II. 20. UTILIZATION OF VERTICAL PROFILE OF DSD INTO BUILDING UP AN ALGORITHM FOR ESTIMATING GROUND RAINFALL AMOUNT USING RADAR

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Introduction

There are many uncertainties in estimating ground rainfall intensity or amount of rainfall using radar. For example anomalous propagation echo, bright band, natural variability of rain drop size distribution (DSD), vertical profile of reflectivity, beam filling by rain drops, conversion of radar reflectivity to rain rate, and random error in reflectivity measurement can be listed (Doviak and Zrnić, 1984; Krajewski, 1993). The purpose of this paper is taking the first step to overcome the issues of natural variability of DSD and also vertical profile of reflectivity. In order to realize this, it is important to obtain both vertical profiles of DSD and radar reflective factor, and to use them in analyzing relations between structures of vertical profiles of DSD and characteristics of rainfall phenomena themselves, including the three-dimensional structures of radar reflectivity.

Firstly, observations of a vertical profile of the DSD using a vertical pointing Doppler radar is shown. Secondly, the observed DSD is used to formulate a dependency of rainfall intensity on the height as a first order approximation. A formulation of the vertical profile of the DSD itself is not included in this paper, since the main concern of this paper is to show that utilization of vertical information on radar reflective factor improves rainfall estimation, even by the use of the first order approximation. Thirdly, an observation equation for estimating ground rainfall using a three-dimensional distribution of radar reflectivity is presented. Another observation equation is also formulated using the width of Doppler spectrum as information on broadening of DSD, however, a practical example is not provided in this paper. Finally, the first observation equation is applied to a volume scanning radar in the NEXRAD network in U.S.A. and to a conventional volume scanning radar in Japan.
Fig 1 - Locations of the MU radar, the Miyama radar and the AMeDAS raingage stations.

OBSERVATION AND ANALYSIS USING MU RADAR

Outline of the Observation

Observations of DSD are currently carried out both by the Disdrometer in conjunction with an optical instrument on the ground surface, and by a vertical pointing VHF Doppler radar in Japan named the MU (Middle and Upper) radar. The MU radar is capable of detecting vertical profiles of the Doppler spectrum which is composed of both the rain drop itself and the air movements (Fukao et. al, 1985). Assuming terminal fall speed of the rain drop to be a function of its diameter, we can derive vertical profiles of DSD from the vertical distribution of the Doppler spectrum observed by the MU radar.

The ground based measurements are carried out at the radar site. Temporal and vertical resolutions of the Doppler spectrum from the MU radar are 3 minutes and 150 m, respectively. The temporal resolution of accumulated information of DSD from the Disdrometer and the Optical instrument is 1 minute. Time series of 1 minute rainfall is also being observed at the radar site by another optical instrument.

The observation domain of the MU radar is covered by a C-band conventional volume scanning radar named the Miyama radar which is operated by the Ministry of Construction of Japan. Figure 1 shows locations of these radars as well as raingage stations of AMeDAS (Automated Meteorological Acquisition System ) managed by the
Japan Meteorological Agency.

Outline of Estimating Vertical Profile of DSD from Doppler Spectra

Because detailed discussion on the computer processing for deriving DSD from the Doppler spectra is documented in Sato et. al [1990], only the principle of the estimation is outlined here.

The Doppler velocity spectrum \( S_p(v) \) which forms the echo due to precipitation by a vertical radar beam in the condition without atmospheric turbulence and wind, is expressed in terms of the diameter of rain drop \( D \) as

\[
S_p(v) = CNMD \frac{dN(D)}{dD}
\]

where \( N(D)dD \) is the number of drops with diameter between \( D \) and \( D + dD \), \( v_p(D) \) is the drop fall velocity for drop-size \( D \) (positive being the upwards direction), and \( C \) is constant. The relation proposed by Gunn and Kinzer [1949] can be used as a function for \( v_p(D) \).

\( N(D) \) is approximated by the following Gamma distribution;

\[
N(D) = \begin{cases} 
N_0D^m \exp(-\Lambda D) & \text{for } (0 \leq v_p \leq v_{max}) \\
0 & \text{for } (v_p > v_{max}, \ 0 > v_p)
\end{cases}
\]

where \( N_0, \Lambda \) and \( m \) are parameters which depend on the type of rainfall. The parameter \( v_{max} \) is introduced so that the existence of a maximum size rain drop \( D_{max} \), say 6 mm or so in diameter, could be taken into consideration.

On the other hand, the Doppler spectrum \( S_t(v) \) due to atmospheric turbulence by a vertical radar beam is approximated by the following Gaussian function;

\[
S_t(v) = R \exp\left[-\frac{(v-w)^2}{2\sigma^2}\right]
\]

where \( w \) is the mean wind velocity in the radar beam direction and \( \sigma \) is the spectral broadening. If the rain drops completely coincide with the motion of atmospheric turbulence, the observed Doppler spectrum \( S(v) \) due to the combination of rain drop and atmospheric turbulence is expressed by

\[
N(v) = S_t(v) + S_p(v) \cdot S_t(v) + P_{rt}
\]
where $S_0(\nu)$ is a normalized form of $St(\nu)$, $Pn$ is the noise level on the spectra and the asterisk denotes the convolution operation.

This theoretical spectrum (2.4) is completely defined by the parameters $P0, w, s$ for background atmosphere, and $N0, m, L, v_{max}$ for rain drops. These eight parameters can be identified by fitting $\log(S_{obs}(\nu))$ to $\log(S(\nu))$ using a nonlinear fitting algorithm. Identification under the assumption $m = 0$ (Marshall Palmer type) is also possible.

Using the above identified parameters and Eq. (2.2), the radar reflective factor $Z$ and the rainfall intensity $Rr$ can be computed by the following equations;

$$ Z = \int_{0}^{D_{mx}} N(D)D^6 dD, \quad R_r = \frac{4}{3} \int_{0}^{D_{mx}} N(D) \nu(D) \left( \frac{L}{2} \right)^3 dL $$

Analysis

Figure 2(a) shows a time series for the vertical distribution of the radar reflective factor computed by Eqs (2.2) and (2.5) (namely, as DSD of Gamma type). Figure 2(b) shows a time series computed using DSD identified under the condition $m = 0$ in Eq. (2.2) (namely, as DSD of the Marshall Palmer type). Because the two images are quite similar, discussion in this paper will focus on the case of DSD identified as the Marshall Palmer type.

![Fig. 2 - Time series of vertical profile of the radar reflective factor computed using DSDs of (a) Gamma type and (b) Marshall Palmer Type.](image-url)
Rainfall was brought on by Typhoon (T9426) which passed right over the MU radar site at 22:44, indicated by the arrow in each image. Because the height of the bright band is about 4.5 km from sea level and the lower limit for the observation by the MU radar to obtain reasonable Doppler Spectrum is about 2 km in height, estimations between the two levels are used here to find the height dependency of rainfall intensity. Figure 3(a) and (b), respectively, shows plots of ratios of 15 minute and 30 minute accumulated rainfalls ($Rr$) estimated by the MU radar to those observed by optical rainage at the ground ($Rg$), depending on the height from sea level.

![Graph](image)

Fig. 3 - Height dependencies of $\ln(Rr/Rg)$ where $Rr$ is rainfall intensity above ground and $Rg$ is on the ground surface. 30 minute averaged values are used in (a) and 60 minute averaged values are used in (b), as $Rr$ and $Rg$.

From this figure it is reasonable to approximate the height dependency by

$$R_r = \alpha_h \cdot R_g \exp(-fh)$$
Here, $h$ is the vertical distance between heights of the radar beam and raingages on the ground. The abias can be interpreted as bias between radar observation and raingage observation.

FORMULATION OF OBSERVATION EQUATIONS FOR ESTIMATION

In the formulation, the drop size distribution is assumed to be the Marshall Palmer type, namely;

$$N(D) = N_0 \exp(-aD^b), \quad \Lambda = aR_r^h.$$  

Here, $R_r$ is the rainfall intensity in [mm/h] at the height of the radar beam, and $a$, $b$ are coefficients to be identified. Then, the radar reflective factor $Z$ can be written as

$$Z = \int_0^\infty D^2N(D)dD = \frac{N_0 b^2}{\Lambda}.$$  

Coefficients relating to the unit transformation are omitted here. In addition, as a relationship between $Rg$ and $Rr$;

$$R_r = aR_g \exp(-bgh),$$

can be assumed to be an first order approximation based on the result shown in the previous Section (page 308). Assuming that $Z$ and $Rg$ can be observed in real-time, an observation equation for the method of least squares,

$$lnZ = lnA_1 + 7\lnR_g - 7ch$$

can be obtained, where $c = bhb$.

Furthermore, the Doppler spectrum width $sv$ is operationally observed using NEXRAD radars. In this case, $sv$ can be used in identifying $L$, a parameter related to the width of the DSD, by assuming that

$$\Lambda = d[f \sigma]^e.$$  

Here, $d$ and $e$ are coefficients to be identified. The function $f$ should be defined so that the elevation angle of the radar beam at the target point could be taken into consideration. It would be reasonable, for example, to assume that the value of $f$ equals 1 when the beam is pointing vertically, whereas it equals 0 when the beam is pointing horizontally. In
focusing on only one beam scan, $f$ could be defined as a function of the beam height both because the elevation angle at the target point monotonically increases as the distance from the radar site increases and because the distance and the height have one to one correspondence. Using Eq. (3.5) together with Eqs (3.2) and (3.3),

$$d(r, f_{3}) = A_{2} - B_{1}n_{R_{g}} + ch$$

can be obtained as an additional observation equation.

In summary, Eq. (3.4) can be used as a single observation equation when only the radar reflective factor $Z$ is available, while Eqs (3.4) and (3.6) can be used in conjunction with when both $Z$ and the Doppler spectrum width $s_{v}$ are available. In this paper, however, the main concern is to show effects of taking the beam height into consideration, thus utilization of $s_{v}$ is only limited to the formulation above.

**CASE STUDY USING NEXRAD**

In our first case study, Eq. (3.4) was applied to the radar reflective factor detected by a radar of the NEXRAD network together with a set of time series of 5 minute rainfall amount observed by nine MESONET stations from 12:00 LST to 15:00 LST on May 29 in 1994 around the Blue River Basin in the State of Oklahoma, U.S.A. The NEXRAD radar used here is the KTLX placed at Norman, Oklahoma, U.S.A. The temporal and horizontal resolutions are about 5 minutes and 1 km. Figures 4 and 5 show the positions of the MESONET stations along with boundary of the Blue River Basin and a horizontal distribution of radar reflectivity detected by the lowest bean scan. The size of the range is 127 km by 117 km. In this case, a rainband composed of convective rainfalls were quickly passing over the basin from the north to the south.

Information only from the lowest beam scan was used in this case study. Even in this case, the dependency on the height can be introduced in identifications and estimations because the beam height monotonically increases as the distance from radar site increase. When considering $Z$ and $R_{g}$ which should be used in Eq. (3.4), averaged values over 5, 10, 15, 30 and 60 minute intervals were separately tested. Here, each time interval is referred to as the accumulation time $T_{ac}$. Moreover, the same intervals are tested at which time, parameters to be identified were assumed to be constant.
This time is referred to as the identification time $T_{id}$. Among all possible combinations of selecting ($T_{ac}$, $T_{id}$), the (30 minute, 60 minute) combination gave the best estimation in terms of the correlation coefficient between observed 5 minute rainfall, $R_g$ and its estimate, $R_e$. Figure 6 shows a comparison between $R_g$ and $R_e$. Figure 6(a) is the result when the parameter $c$ of the last term in the right hand side of Eq.(3.4) is identified, whereas Figure 6(b) is the result when the parameter $c$ is assumed to be 0. In other words, the difference between Figures 6(a) and 6(b) is whether the dependency on height is taken into consideration or not. In this Figure, $r$ represents the correlation coefficient for each 1 hour time period. Improvement to the estimate by introducing the dependency on height can be noticeably seen in the case of overestimation.
Table 1 shows, by correlation coefficients, the accuracies of the estimation when compared with the estimation using 300 and 1.4 as B and b, which is conventionally used in the NEXRAD network in the Z - R relation (namely \( Z = BR^b \)). Table 2 shows observed and estimated total rainfall amount on each MESONET station. Improvement by introducing the dependency on height can be more clearly seen when the accuracy by the conventional method is not high.

**Table 1 - Correlation coefficients of 5 minute rainfall estimations by proposed method and conventional method using a NEXRAD radar**

<table>
<thead>
<tr>
<th>Time</th>
<th>Proposed Method</th>
<th>Conventional Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:00-13:00</td>
<td>0.6604</td>
<td>0.5894</td>
</tr>
<tr>
<td>13:00-14:00</td>
<td>0.8645</td>
<td>0.8667</td>
</tr>
<tr>
<td>14:00-15:00</td>
<td>0.5959</td>
<td>0.5808</td>
</tr>
<tr>
<td>Total</td>
<td>0.7521</td>
<td>0.709</td>
</tr>
</tbody>
</table>

Table 2 Total rainfall amounts observed by MESONET raingages, estimated by proposed method and by the conventional method

<table>
<thead>
<tr>
<th>MESONET Stations</th>
<th>ADA</th>
<th>BYA</th>
<th>CNT</th>
<th>DUR</th>
<th>LANE</th>
<th>MADI</th>
<th>PAUL</th>
<th>SULP</th>
<th>TISH</th>
</tr>
</thead>
<tbody>
<tr>
<td>MESONET Obs</td>
<td>19.26</td>
<td>14.26</td>
<td>30.68</td>
<td>26.69</td>
<td>34.92</td>
<td>32.79</td>
<td>46.17</td>
<td>33.33</td>
<td>24.37</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>7.81</td>
<td>9.63</td>
<td>15.65</td>
<td>23.71</td>
<td>32.5</td>
<td>29.32</td>
<td>16.1</td>
<td>16.77</td>
<td>27.12</td>
</tr>
</tbody>
</table>
As the next case study, a three-dimensional distribution of radar reflective factor observed by the Miyama radar and 1 hour rainfall by the AMeDAS are used. Locations of AMeDAS raingages can be seen in Figure 1. Raingages used in this case study are indicated within the shaded region. Because only 1 hour rainfall amount is available by AMeDAS, the (60 minute, 60 minute) combination is chosen as \((T_{ac}, T_{id})\). In turn, not only information from the lowest beam scan but also that from volume scan are used here to check the usefulness of additionally introducing the three-dimensional distribution. The resolutions of the radar information are 3 km horizontally and 1 km vertically. The operation is composed of two modes of scanning. One is volume scanning from the elevation angle of 1 degree to 23.5 degrees. The other is scanning by a fixed elevation angle of 0.4 degree, which is composed of 5 revolutions. Because more averaged values are obtained by the fixed elevation mode (and therefore, qualities of information from both modes are different), only information from the volume scanning mode was used in this case study. More precisely, because values of \(B\) and \(b\) in the conventional \(Z_nR\) relation for this radar were identified by the fixed elevation mode, estimation using the conventional \(Z_nR\) relation is not compared with estimates using the volume scanning mode. The volume scanning mode takes 4 minutes while the fixed elevation mode takes 1 minute. Therefore the temporal resolution is 5 minutes. Hereafter, the lowest angle beam in the volume scanning mode is referred to as the Lowest Beam.

Figure 7(a) and (b) shows the results in the same manner as Figure 6, using only the Lowest Beam. Namely, Figure 7(b) is the case of neglecting the dependency on height. In the computation of the correlation coefficient, estimations larger than 250 mm/h are excluded because it is noticed here that there were a few abnormally high overestimation. Similar to the case of the NEXRAD, improvement to the estimation by introducing the dependency on height can be distinguished.

![Image](image_url)
assumed to be 0. \( r \) represents the correlation coefficient for each 1 hour time period. Only the Lowest Beam scan is used here.

Figure 8 shows the result when the three-dimensional information below the bright band is considered. Information from the lowest and the second lowest beams are mainly used depending on the distance from the radar site. This indicates that utilization of the three-dimensional information further improves the accuracy from the case of using only beam with the smallest elevation angle.

![Graph comparing AMeDAS Rg and estimated Re](image)

**Fig. 8** - Comparison between \( Rg \) and \( Re \). \( r \) represents the correlation coefficient for each 1 hour time period. Three-dimensional distribution of radar reflectivity below the bright band is used.

**CONCLUSION**

In conclusion, this paper showed the followings:

- Observations of DSD by a vertical pointing VHF Doppler radar in Japan named the MU (Middle and Upper) radar.

- Vertical profile of logarithm of rainfall intensity estimated using the observed DSD have a linear dependency on height as first order approximation.

- An observation equation is formulated based on DSD of the Marshall Palmer type and the dependency of rainfall intensity on the height.

- Another observation equation is also formulated using the width of the Doppler spectrum as information on the width of DSD.

- The first equation was applied to information from a NEXRAD radar, which used a single elevation beam scan. In result the introduction of the height dependency
improved accuracy of estimates.

- The first formulation was applied to information from a Japanese radar, as an application to information from volume scan. In result the utilization of three-dimensional distribution of radar reflective factor further also improved accuracy of estimates.

In this paper, the observed vertical profile of DSD was used only to obtain the height dependency of rainfall intensity. A logical next step would be to recreate observation equations in terms of vertical profile of the parameters of DSD itself. On the other hand, since it was found that a few abnormally high overestimation existed in the applications (although it was not strongly mentioned so far), it can be argued that they result from the utilization of the logarithmic expressions in the observation equation (because estimated values are quite sensitive to the identified parameter). Therefore, another next step would be introducing a nonlinear identification procedure without using an observation equation in logarithmic form. In spite of these two recommended steps, it can be said this paper showed effectiveness in utilizing the three-dimensional distribution of radar reflective factor for estimating good rainfall amount.

REFERENCES


