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Estimation error of spectral parameters of mesosphere-stratosphere-troposphere radars obtained by least squares fitting method and its lower bound

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We have calculated the estimation error of parameters of echo power spectra observed by mesosphere-stratosphere-troposphere (MST) radars by means of computer simulations for least squares fitting and moment methods. The least squares fitting method is shown to be better than the moment method in the region with low signal-to-noise ratio (snr), especially for narrow spectra. However, the estimation error of the fitting method at infinite snr is approximately twice that of the moment method. This has been attributed to the nature of the statistical fluctuation of the power spectral density, which shows a χ^2 distribution. For both methods at infinite snr we have derived equations which show the accuracy of the estimates versus observation period and spectral width. When we use the fitting method for the data observed with the MU radar (46.5 MHz), the typical errors of the radial wind velocities are 0.7 and 2.0 m s^{-1} in the stratosphere and in the mesosphere, respectively. By calculating the logarithm of the spectrum with inifite snr and fitting a parabolic curve to it, the error of the Doppler shift has become approximately 20 times smaller than that of the moment method. It has been shown that this is one technique to achieve the theoretical lower bound of the estimates.

1. INTRODUCTION

In mesosphere-stratosphere-troposphere (MST) radar observations there are several techniques to estimate parameters such as echo power, radial wind velocity, and spectral width. For the return signal of MST radars we can assume that a power spectrum of the radar returns shows a Gaussian distribution which is described as follows [e.g., *Woodman*, 1985]:

$$S(f) = \left[\frac{P}{(2\pi)^{1/2}\,\sigma}\right] \exp\,[-(f-f_d)^2/2\sigma^2] \qquad (1)$$

where f is frequency and P, f_d, and σ are echo power, mean Doppler shift, and spectral width, respectively.

One of the techniques to determine these parameters is a moment method. The zeroth, first, and second moments of the spectral density correspond to the echo power, mean Doppler shift, and spectral width, respectively. For an observed spectrum $S'(f_i)$ obtained at M discrete frequencies f_i,

$$P = \sum_{i=1}^{M} S'(f_i) \qquad (2)$$

$$f_d = \frac{1}{P} \sum_{i=1}^{M} f_i S'(f_i) \qquad (3)$$

$$\sigma^2 = \frac{1}{P} \sum_{i=1}^{M} (f_i - f_d)^2 S'(f_i) \qquad (4)$$

Another technique is a least squares fitting method. A Gaussian spectrum is fitted to the observed one so as to minimize the squared sum of the residual

$$\varepsilon^2 = \sum_{i=1}^{M} [S'(f_i) - S(f_i;\, P, f_d, \sigma)]^2 \qquad (5)$$

by changing the parameters P, f_d, and σ. The spectral parameters may also be estimated from the autocorrelation function of the time series of the radar returns. The radial velocity is estimated from the phase angle of the autocorrelation function at the first lag [e.g., *Woodman and Guillén*, 1974]. *Sato and Woodman* [1982] applied a least squares fitting technique to the autocorrelation at multiple delays when radar returns consist of a component from fading ground clutter together with the turbulence echo.

The performance of many estimators were compared by *Zrnić* [1979] and *Woodman* [1985], and the performance of the pulse pair method was shown to

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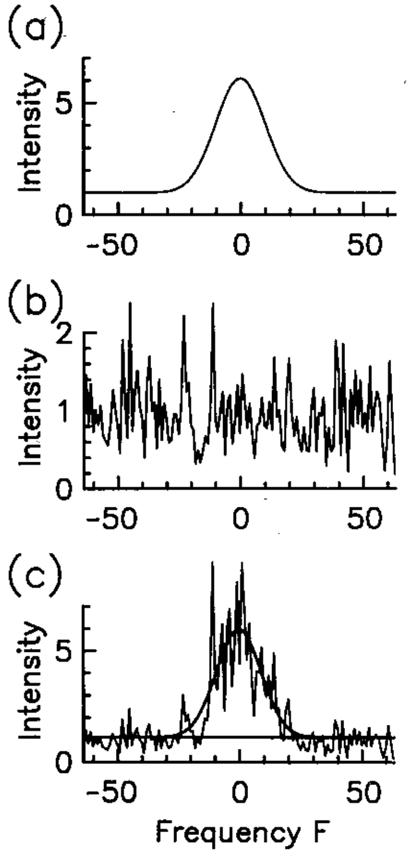

Fig. 1. An example of (*a*) Gaussian spectrum, (*b*) statistical fluctuation, and (*c*) model spectrum utilized in the computer simulation. Frequency F is normalized by the frequency interval between the discrete spectral components. Signal-to-noise ratio, spectral width, number of incoherent integrations, and noise level of the model spectrum are $S_N = 1$, $W = 10$, $n = 5$, and $P_N = 1$, respectively.

be better than that of the moment method at low signal-to-noise ratio (snr) although they are the same at infinite snr. *Zrnić* [1979] also showed the theoretical minimum estimation error (Cramer-Rao bound) of spectral parameters which is obtained by maximum likelihood (ML) estimators. However, there are no theoretical calculations for the performance of the fitting method, which may be expected to show better results in comparison to other techniques, especially in the region with low snr.

The MU radar (35°N, 136°E), which has been operated since 1983, is a monostatic pulse Doppler radar with a carrier frequency of 46.5 MHz [*Kato et al.*, 1984; *Fukao et al.*, 1985*a*, *b*]. The received signal is transformed to Doppler power spectra by using the fast Fourier transform (FFT) procedure. For parameter estimation the fitting method is utilized in almost all of the MU radar observations. In this paper we discuss the performance of the fitting method by using the computer simulation technique and compare it with the moment method. We also investigate problems which may occur with the fitting method when it is applied to the power spectrum of a random process and show a possibility to improve the performance of the fitting method and to approach the Cramer-Rao bound.

2. MODEL SPECTRUM AND COMPUTER SIMULATION TECHNIQUE

As shown in Figure 1*a*, we have calculated a power spectral density of $S(f_i) + P_N$ at 128 discrete frequencies f_i, where P_N is noise level density. For convenience the Doppler shift f_d is assumed to be zero. The snr is defined as the power ratio between the signal and the noise as follows:

$$S_N = P/MP_N \tag{6}$$

where $M = 128$ is the number of discrete frequencies. In the following sections we use normalized frequency and spectral width

$$F = f/\Delta f \tag{7}$$

$$W = \sigma/\Delta f \tag{8}$$

where Δf is a frequency interval between the discrete spectral components.

The output of the receiver, which is a time series of data, is a random process with a Gaussian distribution. Because the Fourier transform is a linear transformation and power spectral density is a squared sum of both real and imaginary parts of the spectral component, the power spectral density has a statistical fluctuation with a χ^2 distribution [e.g., *Bendat and Piersol*, 1971]. The statistical fluctuation of the spectrum is simulated by generating random numbers with χ^2 distribution as shown in Figure 1*b*. The model power spectrum is calculated as a product of the Gaussian spectrum and the statistical fluctuation (Figure 1*c*). The amplitude of the statistical fluctuation can be reduced by averaging successive power spectra, which is called incoherent integration. The standard deviation of the model spectrum is proportional to the spectral density itself and is equal to $S(f_i)/\sqrt{n}$, where n is the number of the incoherent integration. The fluctuation shown in Figure 1*b* corresponds to $n = 5$. It should be noted that the model spectrum does not include the effects of a windowing

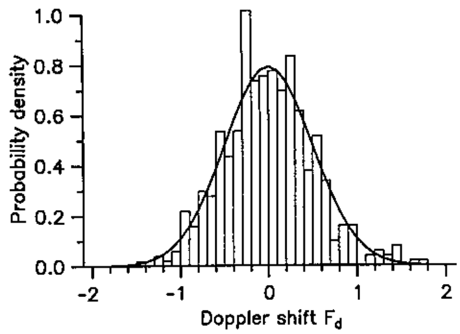

Fig. 2. Distribution of 500 Doppler shift F_d estimates obtained by the fitting method. The simulation condition is $S_N \rightarrow \infty$, $W = 3$, and $n = 5$. Solid curve corresponds to the Gaussian distribution with the bias and standard deviation of the estimated F_d.

function, which may bias the spectral width estimates, especially for narrow spectra.

Spectral parameters are estimated for each spectrum by using both the moment and the least squares fitting method. First, the generated spectrum is smoothed by using a numerical low-pass filter with a −3-dB cutoff period of 20Δf. We find the frequency point with the largest spectral density in the smoothed spectrum, which is used as the first guess of the Doppler shift. The low-pass filter consists of a running mean with the von Hann window. The goodness of the initial guess depends largely on the shape of the window function, especially for poor snr. The von Hann window used here gives substantially better estimates than simple boxcar or triangular windows. After subtracting the real P_N from the original spectrum, we have applied both techniques to 64 spectral points around the first guess.

An additional difference between the two techniques is that we have used all spectral points above and below the noise level in the fitting method, whereas only positive spectral points are used in the moment method. The smoothed curve in Figure 1*c* is an example of the spectrum estimated by the fitting method. It should be noted that the performance of both the fitting and the moment methods relies on the goodness of the initial guess of the Doppler shift. Actually, the low-pass filter operation involved in deriving the initial guess takes more computer time than the fitting or the moment method themselves.

In order to obtain the estimation errors of parameters, we have calculated 500 model spectra and obtained the bias and variance of the estimates. Figure 2 shows a distribution of the Doppler shift determined by the fitting method. We use a normalized Doppler shift $F_d = f_d/\Delta f$ in this figure. The solid curve is a Gaussian distribution which has a bias b_F and variance σ_F^2 of F_d. Because the number of samples is large enough, the distribution is close to Gaussian. We define the estimation error of the Doppler shift, E_F, as follows:

$$E_F = (\sigma_F^2 + b_F^2)^{1/2} \tag{9}$$

This is called the "rms (root mean square) error" [*Bendat and Piersol*, 1971]. E_F is 0.51 in this case.

Estimation errors of the spectral width and the echo power are also defined in the same way as shown in (9) and are represented by E_W and E_P, respectively. E_W is normalized by Δf, which is the same procedure as for E_F. On the other hand, E_P is described as the ratio to the true echo power P.

3. ESTIMATION ERROR AT FINITE SIGNAL-TO-NOISE RATIO

Figure 3 shows E_F for both the fitting and moment methods versus S_N when $n = 5$. This case corresponds to the estimation error when we averaged five spectra and then applied both methods. When $W = 2$ for the fitting method, E_F is 0.44 in the region with large S_N. The fitting is successful at $S_N > 10^{-1}$, but we recognize a rapid increase of E_F below this level. The wider spectrum causes a larger estimation error of the Doppler shift; e.g., $E_F = 0.91$ for $W = 10$ at $S_N = 10^4$. However, E_F starts increasing at a similar S_N to that for $W = 2$. It is noted that estimation errors are almost constant with S_N above this level. Comparison of the results with that of the moment method shows that at high S_N the estimation error of

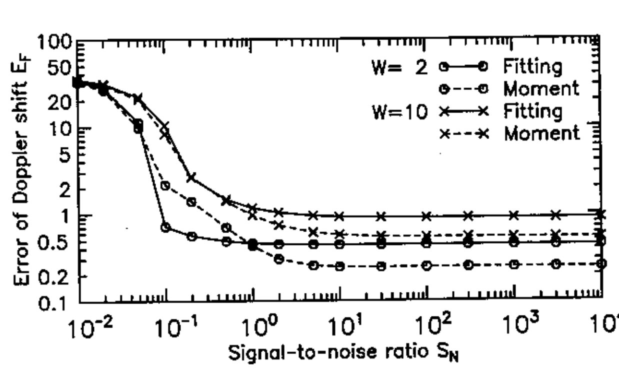

Fig. 3. Variation of the estimation error of the Doppler shift versus signal-to-noise ratio for $n = 5$. Circles and crosses show results for $W = 2$ and $W = 10$, respectively. Solid and dashed lines correspond to the errors obtained by the fitting and moment methods, respectively.

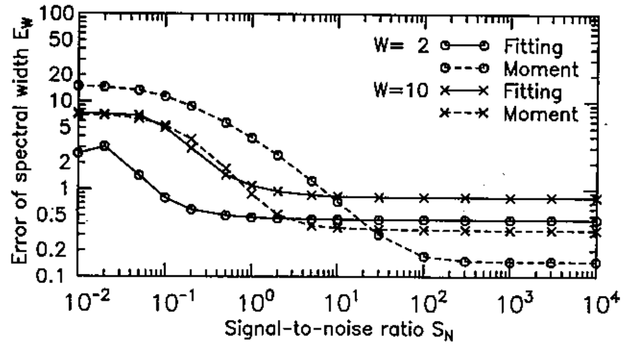

Fig. 4. The same as Figure 3, except for the estimation error of the spectral width.

the moment method becomes less than that of the fitting method; at $S_N = 10^4$, $E_F = 0.25$ and 0.52 for $W = 2$ and 10, respectively. The estimation error of the moment method is approximately 60% of the fitting method in this region. As S_N decreases, the estimation error of the moment method gradually increases. The error of the moment method at $10^{-1} < S_N < 1$ are almost the same as that of the fitting method for $W = 10$ and worse for $W = 2$. There is no significant bias shown in the low-S_N region. Below $S_N = 10^{-1}$, both methods show a rapid increase of the estimation error. The large error is obtained because the numerical filter captures the highest peak randomly distributed in the fluctuating noise level and accepts it as the first guess of the Doppler shift. When we assume a constant distribution of E_F within the spectral window of $F = \pm 64$, the standard deviation is approximately 37, which is consistent with E_F at $S_N = 10^{-2}$.

Figure 4 illustrates the estimation error of the spectral width E_W. At large S_N with $W = 2$, E_W is 0.45 and 0.16 for the fitting and moment methods, respectively. Better results by the moment method are also obtained with $W = 10$. E_W of the moment method is about 40% of the fitting method. E_W of both methods does not decrease very much with increasing S_N at $S_N > 10^2$. These results are similar to those of the estimation error of the Doppler velocity. In the low-S_N region, W is well estimated for $S_N > 1$ by using the fitting method. The performance of the moment method is not good at $S_N < 10$. E_W of the moment method is much worse than that of the fitting method for $W = 2$, although E_W of the moment method for $W = 10$ is not larger than that of the fitting method at any S_N. The increase of E_W in the low-S_N region for both methods is due to the larger bias. For the fitting method, around $S_N = 10^{-2}$, the estimated spectral width is smaller than the true width, and $W \sim 1$ or 2. This is because the fitting routine captures the major peak of the statistical fluctuation and the estimated width is locked to the width of the spikes in the fluctuating noise. On the other hand, the spectral width obtained by the moment method is larger than the true width, because the distribution of the spectral density becomes close to the uniform distribution in the spectral points used in the calculation.

As shown in Figure 5, the relative estimation error of the echo power has a different nature from E_F or E_W. When $S_N > 1$, the results obtained by both methods are close and almost constant with increasing S_N. E_P is better for larger W. In the high-S_N region the relative estimation errors are approximately 17 and 8% of the true P when W is 2 and 10, respectively. This is because we can average more independent spectral points for the wide spectrum than for the narrow spectrum.

Comparison of the results obtained by the fitting and moment methods shows that the performance of the moment method is better than the fitting method at high S_N. At $S_N < 1$, on the other hand, E_W of the fitting method is smaller than that of the moment method for $W = 2$. E_F of the fitting method also shows slightly better results for the narrow spectrum in the low-S_N region. As cited above, large E_W of the moment method at low S_N is due to the large bias, which is determined by the number of spectral points used in the calculation. E_W of the moment method can be improved by selecting the spectral points differently. However, we need to know the range of the spectral width prior to the calculation. The fitting method is less sensitive to the number of the spectral points and is the safer estimator for the spectral width.

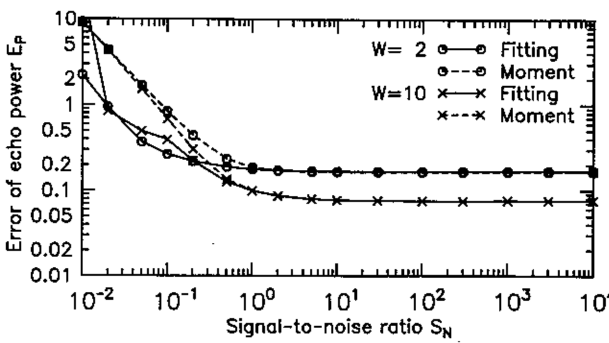

Fig. 5. The same as Figure 3, except for the estimation error of the echo power.

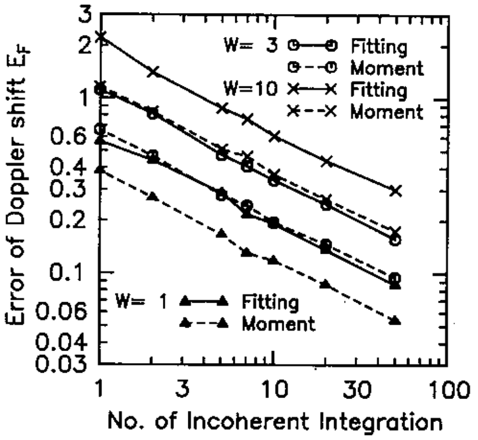

Fig. 6. Estimation error of the Doppler shift versus number of incoherent integration of the spectra at inifite S_N. Triangles, circles, and crosses correspond to $W = 1$, $W = 3$, and $W = 10$, respectively. Solid and dashed lines correspond to the errors for the fitting and moment methods, respectively.

4. ESTIMATION ERROR AT INFINITE SIGNAL-TO-NOISE RATIO

Since the estimation errors of parameters do not vary much according to S_N for high-S_N cases, the estimation error at infinite S_N is a good index for the errors in real observations. Figure 6 shows the estimation error of the Doppler velocity versus number of incoherent integrations of the spectra. The estimation error of the Doppler shift is almost proportional to $\sqrt{W}$ and inversely proportional to $\sqrt{n}$. For the spectral width, as illustrated in Figure 7, the response of E_W to n and W is the same as that for the

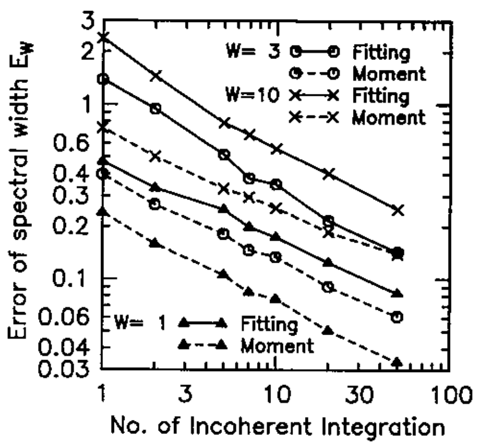

Fig. 7. The same as Figure 6, except for the estimation error of the spectral width.

TABLE 1. Coefficient k of (10) for Estimation Errors of Normalized Doppler Shift and Spectral Width Obtained Using Moment and Fitting Methods

	Fitting	Moment
F_d	0.63	0.38
W	0.60	0.24

Doppler shift. At infinite S_N the estimation error E of either F_d or W can be described as

$$E = k(W/n)^{1/2} \tag{10}$$

where k is a constant. By using the data shown in Figures 6 and 7, we have determined the constant k for both methods and spectral parameters, which are listed in Table 1.

In order to determine the estimation error of the radial wind velocity obtained by using both methods, we put physical dimensions to the normalized estimation error E_F. The estimation error ε_v of radial wind in the unit of meters per second is

$$\varepsilon_v = (c/2f_0T)E_F = (\lambda/2T)E_F \tag{11}$$

where c (meters per second), f_0 (hertz), λ (meters), and T (seconds) are the speed of light, the carrier frequency of the radar, the wavelength of the radar, and the length of the time series of the data, respectively. The spectral width σ_v in the unit of meters per second is also described as follows:

$$\sigma_v = (\lambda/2T)W \tag{12}$$

By substituting (10) and (12) into (11),

$$\varepsilon_v = K(\sigma_v/T_0)^{1/2} \tag{13}$$

where

$$K = k(\lambda/2)^{1/2} \tag{14}$$

is a constant and

$$T_0 = nT \tag{15}$$

is the total observation period to obtain a set of spectral parameters. Equation (13) shows that if S_N is infinite, the observation period is the only factor that we can choose when we observe the radial velocity by using the radar. The spectral width in this case should contain all of the broadening effects such as beam and shear broadening besides the spectral broadening causing by the turbulence. We should note that (13) is also valid for the estimation error of the spectral width in the unit of meters per second.

TABLE 2. Coefficient K of (14) for Estimation Errors of Radial Wind Velocity and Spectral Width in the Unit of Meters per Second Obtained Using Moment and Fitting Methods

	Fitting	Moment
Radial velocity	1.1	0.67
Spectral width	1.1	0.43

We have calculated the coefficients K for the estimation errors expected in the MU radar observations by using k in Table 1 and $\lambda = 6.45$ m ($f_0 = 46.5$ MHz) and show them in Table 2. When we observe the radial wind velocity every 1 min by using the fitting method, $\varepsilon_v = 0.12$ m s^{-1} for $\sigma_v = 0.7$ m s^{-1} which is typical spectral width in the stratosphere, and $\varepsilon_v = 0.20$ m s^{-1} for $\sigma_v = 2.0$ m s^{-1} which is typical in the mesosphere. For the moment method, *Zrnić* [1979] has theoretically calculated the estimation error of both radial velocity and spectral width. The theoretical formulas show that with the MU radar and at infinite S_N, $K = 0.68$ and $K = 0.41$ for radial wind velocity and spectral width, respectively. These values are consistent with those obtained by the simulations.

We infer that the larger errors with the fitting method occur because of the nature of the statistical fluctuations of spectral coefficients. Because the amplitudes of the statistical fluctuations are proportional to the spectral density, components around the spectral peak have larger fluctuations than the spectral components with low spectral density. Thus the sum of the squared residual is almost solely determined by the spectral components around the peak, which implies that we actually use only a portion of the spectrum when we estimate the parameters by the fitting method.

5. LOWER BOUND OF THE SPECTRAL PARAMETER ESTIMATION

In order to investigate the performance of both methods at infinite S_N, we have calculated the normalized residual of the estimated spectral density to the model by subtracting the model spectra $S'(F)$ from the estimated spectra $S(F; P, F_d, W)$ as follows:

$$R(F) = (1/p)[S(F; P, F_d, W) - S'(F)] \quad (16)$$

where p is the true peak power density. For the results with the moment method we assume the Gaussian spectrum with the estimated parameters and calculate $R(F)$. The solid line in Figure 8 shows $\langle R(F) \rangle$ for the fitting method, where $\langle\,\rangle$ shows an average over 500 model spectra, and the model parameters are $S_N \rightarrow \infty$, $W = 3$ and $n = 5$. If the parameter estimation is successful, $\langle R(F) \rangle$ should be zero at every F. The expected statistical fluctuation of $\langle R(F) \rangle$ is 2%, because the substantial number of incoherent integration becomes 2500 when $R(F)$ is averaged over 500 model spectra. We recognize a systematic variation of $\langle R(F) \rangle$ for the fitting method. The residual is approximately 2.5% larger than the true spectral density at $F = 0$, and 4% smaller at $F = \pm 4$. The positive $\langle R(F) \rangle$ means that the estimated spectral component is larger than the true one. Because the residual is positive at the center and negative near the shoulder of the initial spectrum, the spectra are estimated to be narrower than the real spectrum. A small negative bias is found in the distribution of the estimated W. The symmetrical nature of $\langle R(F) \rangle$ shows that there is no bias in the Doppler shift estimates. The dashed line in Figure 8 shows $\langle R(F) \rangle$ for the moment method, which is less than $\pm 2\%$ at every F. The amplitude of the fluctuation is less than what is expected from purely statistical fluctuations. The moment method does not show any significant bias to spectral parameters.

The least squares fitting method gives maximum likelihood (ML) estimates when the statistical fluctuations of the samples at different frequency points are uncorrelated and the samples at each frequency point have Gaussian distribution with known variance [e.g., *Brandt*, 1970]. The second condition is not satisfied in the least squares fitting applied to the power spectrum. We use a uniform weight for all spectral components, although the variance of the spectral components is proportional to the spectral density

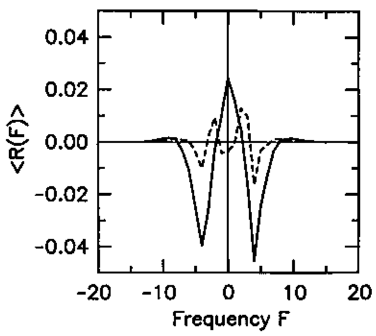

Fig. 8. Residual of the spectral coefficient estimates averaged over 500 model spectra. Solid and dashed lines correspond to the result for the fitting and moment methods, respectively.

itself. Also, the statistical fluctuation shows the χ^2 distribution. *Waldteufel* [1976] has pointed out that the least squares fitting for the radar Doppler spectra does not yield the ML estimates. Figure 9 illustrates an example of the estimated spectrum determined by the fitting method. The model spectrum is normalized by the true peak power density of the spectrum. The χ^2 distribution is not symmetrical with respect to mean of the spectral coefficient and tends to have larger spikes on the higher side in its distribution. The spikes of the statistical fluctuations which appear around the peak of the Gaussian spectrum cause a large bias to the sum of the squared residual and set the fitted spectrum higher at the center and narrower than the true width. The estimation of the Doppler shift is also affected by the positions of the spikes.

Since the amplitude of the statistical fluctuation is proportional to the spectral density itself, we can equalize the amplitude of the statistical fluctuation by taking the logarithm of the spectral density. The variance of the samples at different frequency points becomes constant, although its statistical distribution is not Gaussian. When $S_N \rightarrow \infty$, we can estimate the spectral parameters by fitting a parabolic curve to log $[S(F)]$ with a uniform weight. As shown by the dashed line in Figure 9, the result of this method fits

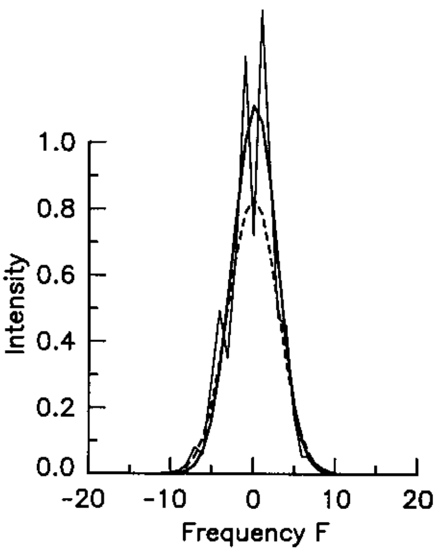

Fig. 9. Examples of the Gaussian spectrum estimates at $S_N \rightarrow \infty$, $W = 3$, and $n = 5$. The heavy solid line and the dashed line show the results estimated by the fitting in the linear domain, and the parabolic fitting in the logarithmic domain, respectively. The light solid line shows the model spectrum normalized by the true peak power density.

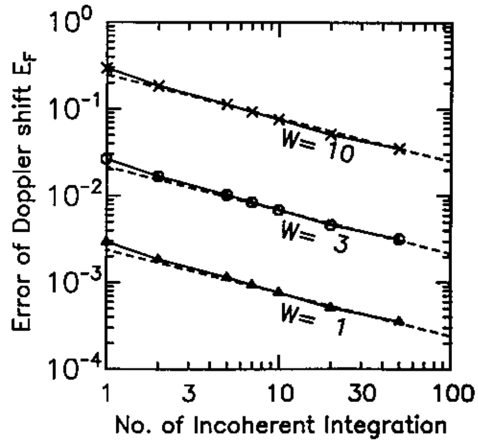

Fig. 10. The same as Figure 6, except for the estimation error of the Doppler shift obtained by the parabolic fitting in the logarithmic domain. Dashed lines show the Cramer-Rao bound of the ML estimator shown by *Zrnić* [1979].

the model spectrum much better than the fitting in the linear domain in components with low spectral density. We have simulated the estimation error of the Doppler shift for this method. The result of this parabolic fitting is shown in Figure 10. The estimation error obtained by this method is, for example, approximately 20 times less than that by the moment method when $W = 3$. The dashed line is the theoretical lower bound (Cramer-Rao bound) obtained for the expected error of the ML estimator [*Zrnić*, 1979]. The estimation error of the normalized Doppler shift agrees very well with the Cramer-Rao bound. The estimation error is inversely proportional to $\sqrt{n}$. Also, we recognize that it is almost proportional to W^2. This is different from the $\sqrt{W}$ dependence shown in (10) but agrees well with the theoretical formula given by *Zrnić* [1979]. The improvement of the estimation error obtained by this fitting in the logarithmic domain is larger for spectra with narrow width than for spectra with larger width.

In theoretical evaluations of the accuracy of the pulse pair or moment method, *Doviak and Zrnić* [1984] and *Woodman* [1985] show the accuracy to be comparable to the Cramer-Rao bound. This is because they replaced the sampling interval by the signal correlation time when they calculate the Cramer-Rao bound. They have mentioned that a sampling faster than the correlation time is redundant and only introduces higher-frequency components with small spectral density. However, for a Gaussian spectrum with infinite S_N, every spectral component should have significant information no matter how far the component is from the spectral peak. This implies that we can arbitrarily increase the number of independent points in the spectrum and improve the performance of estimators as much as we wish. The performance of the moment and fitting methods in the linear domain is restricted, because the equivalent number of effective spectral components is limited around the spectral peak according to the spectral width. The result clearly shows that an infinite S_N, even very high frequency components of the signal spectrum with very small amplitude have substantial importance and can be used to derive its spectral parameters. In the linear domain, contributions from these components are masked by the much larger fluctuations of the frequency components around the peak. Our simulation shows that the parabolic fitting in the logarithmic domain is one technique to realize the lower bound of the ML estimator by making use of such high-frequency components. The detailed discussion of the ML estimator for the Doppler spectra is given by *Waldteufel* [1976]. Waldteufel showed a fitting method to maximize the logarithm of the maximum likelihood ratio itself.

The disadvantage of the parabolic fitting is in the accuracy of the echo power estimation. The dashed line in Figure 9 shows that the peak spectral density is approximately 10% less than the true value, which is a quite large error considering the excellent performance in the estimation of the Doppler shift. This underestimate comes from the fact that in estimating the echo power, we are basically finding the mean in the logarithmic domain. The mean in the logarithmic domain corresponds to the geometric mean, which is always less than or equal to the arithmetic mean in the linear domain.

Also, this method is not realistic for real data with finite S_N. As cited by *Zrnić et al.* [1977], the logarithmic fit is better if spectra are free of artifacts. However, the spectra are contaminated and distorted by truncation distortion, aliasing effects, quantization error, and so on. The estimation error of the noise level will also affect the result. However, if we produce a fitting routine for log $[S(F)]$ which contains these contaminations, it should show much better performance than the existing methods.

6. CONCLUDING REMARKS

In this paper we have investigated the performance of spectral parameter estimators by computer simulations. We have compared the performance of the fitting and the moment methods. The comparison showed that the moment method was better than the fitting technique in the large-S_N region. Also, we have found that the fitting method shows the bias toward slightly narrower spectral width estimates. Both the poor performance and the spectral width bias of the fitting method is attributed to the nature of the statistical fluctuations of the spectral estimates. Because the amplitude of the statistical fluctuation is proportional to the spectral density, parameters are determined mainly by the spectral points around the peak. However, the fitting technique showed better results in the low-snr region, especially for the narrow spectrum. In order to estimate the spectral width in this region the fitting technique is the safer estimator, because the spectral width obtained by the moment method tends to show large positive bias to the true width. The other advantage of the fitting method over the moment method is that the set of parameters is not limited to P, f_d, and σ. When the spectral distribution is not Gaussian, or the spectra are contaminated by other spectral components such as the fading ground clutter, the fitting technique can estimate spectral parameters without bias if the model spectral function can be evaluated correctly [*Woodman*, 1985].

In order to improve the estimation errors of the spectral parameters we should use the moment method when S_N is large enough to produce better results. We infer that this switching between both methods is a good compromise for observations in the stratosphere. For the mesospheric observation, however, the fitting method shows an adequate performance, because S_N is usually less than 10 in the mesosphere with the MU radar.

As shown in section 5 the performance of both fitting and moment methods is much worse than the Cramer-Rao bound expected by using a ML estimator. This is because they utilize only a restricted portion of the spectral components around the peak to determine spectral parameters. Although the theory does not show the algorithm to derive the best performance of the ML estimator, we could obtain the minimum estimation error of the Doppler shift in an ideal case by calculating the logarithm of the model spectra and fitting a parabolic curve to this. However, this technique is not realistic for the spectra with finite S_N, since the spectra show systematic distortions. It is possible to increase the performance of the estimators by producing a fitting technique for log $[S(f)]$ taking these distortions into account.

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REFERENCES

Bendat, J. S., and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, pp. 189–193, Wiley-Interscience, New York, 1971.

Brandt, S., *Statistical and Computational Methods in Data Analysis*, pp. 170–173, North-Holland, New York, 1970.

Doviak, R. J., and D. S. Zrnić, *Doppler Radar and Weather Observations*, pp. 113–115, Academic, San Diego, Calif., 1984.

Fukao, S., T. Sato, T. Tsuda, S. Kato, K. Wakasugi, and T. Makihira, The MU radar with an active phased array system, 1, Antenna and power amplifiers, *Radio Sci., 20*(6), 1155–1168, 1985*a*.

Fukao, S., T. Tsuda, T. Sato, S. Kato, K. Wakasugi, and T. Makihira, The MU radar with an active phased array system, 2, In-house equipment, *Radio Sci., 20*(6), 1169–1176, 1985*b*.

Kato, S., T. Ogawa, T. Tsuda, T. Sato, I. Kimura, and S. Fukao, The middle and upper atmosphere radar: First results using a partial system, *Radio Sci., 19*(6), 1475–1484, 1984.

Sato, T., and R. F. Woodman, Spectral parameter estimation of CAT radar echoes in the presence of fading clutter, *Radio Sci., 17*(4), 817–826, 1982.

Waldteufel, P., An analysis of weather spectra variance in a tornadic storm, *NOAA Tech. Memo ERL NSSL-76*, 80 pp., Natl. Severe Storms Lab., Norman, Okla., 1976.

Woodman, R. F., Spectral moment estimation in MST radars, *Radio Sci., 20*(6), 1185–1195, 1985.

Woodman, R. F., and A. Guillén, Radar observations of winds and turbulence in the stratosphere and mesosphere, *J. Atmos. Sci., 31*(2), 493–505, 1974.

Zrnić, D. S., Estimation of spectral moments for weather echoes, *IEEE Trans. Geosci. Electron., GE-17*(4), 113–128, 1979.

Zrnić, D. S., R. J. Doviak, and D. W. Burgess, Probing tornadoes with a pulse Doppler radar, *Q. J. R. Meteorol. Soc., 103*(438), 707–720, 1977.

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