Power Spectra of Oblique Velocities in the Troposphere and Lower Stratosphere Observed at Arecibo, Puerto Rico

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ABSTRACT

Wind profiles measured with the Arecibo Observatory 430 MHz radar during 1979 and 1980 have been used to calculate frequency and radial wavenumber power spectra. Periods between 2 min and 6 h and vertical wavelengths between 300 m and 13.5 km are covered by the spectra. The data are line-of-sight velocity components obtained with a beam pointing between 5.4° and 15° off-vertical. The presented evidence supports the conclusion that the dominant contribution to the spectra at periods less than 1 h is from the vertical velocity component. The frequency spectra have spectral slopes near -1 for periods of less than 1 h, and the radial wavenumber spectra have slopes between -1 and $-\frac{3}{2}$. The results are discussed in the context of two-dimensional turbulence and a universal gravity-wave spectrum. We also present the spectra from an event associated with convection in the early evening hours. A strong peak in the spectral energy near the Brunt–Väisälä frequency was present during the convective activity. The energy content at longer periods was found to be enhanced in the ensuing period when the convective energy input at shorter time scales near the Brunt–Väisälä period had diminished.

1. Introduction

The article by Gage (1979) revitalized interest in mesoscale power spectra. Prior to that, during the 1960s and in the early 1970s, investigations of mesoscale turbulence had focused on the so-called "mesoscale gap" (e.g., see Atkinson, 1981, chapter 1, for a discussion). A consensus emerged that power was either distributed monotonically between the synoptic scale and the microscale, with no evidence of a mesoscale gap, or that there was actually an enhancement in the power at the small-scale limit of the mesoscale range, presumably due to either convection or shearing instabilities. The region on the large-scale side of the hump was mistakenly interpreted in some cases as being indicative of a lack of power in that range rather than an enhancement above the typical level in the high wavenumber or high frequency range.

Investigations in the 1960s and 1970s showed that spectra of the horizontal velocity as a function of frequency or horizontal wavenumber generally followed a $k^{-5/3}$ power law (e.g., Mantis, 1963; Kao and Woods, 1964; Atkinson, 1981). At the end of the 1970s, new wind measurements with high-resolution balloon tracking and using the wind profiler technique were analyzed by Gage (1979). His power-spectra and structure-function calculations produced results in agreement with a $k^{-5/3}$ power law characteristic of an inertial subrange. However, as Gage argued, simple scaling excluded truly three-dimensional turbulence in the mesoscale range, but a two-dimensional turbulent process could explain the observations if an energy source was located at the small-scale end of the spectrum and energy was propagating along the spectrum toward larger scales.

Larsen et al. (1982) and Balsley and Carter (1982) analyzed more extensive data sets covering height ranges in the troposphere, lower stratosphere, and even the mesosphere, in the latter study. Spectra of winds measured over a month or more were calculated, and the slope for time scales from a few hours up to one day was found to be consistently near $-\frac{5}{3}$. Thus, the value of the slope was well known, but the dynamics responsible for it was still open to interpretation.

Lilly (1983) extended Gage's (1979) arguments with an analytic treatment that showed that waves and turbulence could coexist in the mesoscale range. Lilly postulated that the source of energy for the red cascade of energy from small to large scales was convection. In particular, there may be an analogy between the collapse of wakes to form a stratified flow, as described by Lin and Pao (1979), and the collapse of three-dimensional cloud motions into a stratified flow that is characteristically two-dimensional.

VanZandt (1982) offered an alternative interpretation of the experimental results. He pointed out that the observed power spectra could be the result of a spectrum of gravity waves in analogy with the process that is well known by oceanographers in the form of the Garrett–Munk spectrum (1975). In the oceans, the horizontal velocity/frequency spectra follow a -2 power law, unlike the atmospheric mesoscale spectra, which have a $-\frac{5}{3}$ slope, but the exact value of the slope may not be critical for the universal gravity-wave spectrum interpretation. More crucial is the relationship between the slopes of the spectra of the different velocity components and thermodynamic variables as a function of horizontal and vertical wavenumber and frequency. If we follow the development by Garrett and Munk (1975), then the various spectral slopes can be predicted once a few parameters are specified (see also VanZandt, 1982).

The universal gravity-wave spectrum theory has had success in explaining the frequency and wavenumber spectra of horizontal velocities. Given that the slope of the frequency spectrum is very close to $-\frac{5}{3}$, we expect the horizontal wavenumber spectrum to have nearly the same slope, as discussed by VanZandt (1982). Two-dimensional turbulence will also lead to the same frequency and wavenumber slope, if the Taylor hypothesis is applicable. A subtlety in the gravity wave spectrum is the steepening slope at higher wavenumbers, as shown in Fig. 2 of VanZandt's (1982) article. Nastrom and Gage (1983), Lilly and Petersen (1983), Nastrom et al. (1984), and Nastrom and Gage (1985) have calculated the mesoscale wavenumber spectra directly from aircraft wind data and have found the $k^{-5/3}$ power law to give a good fit to the data, except during "turbulent" episodes when the high wavenumber end of the spectrum attains a shallower slope at scales less than 100 km.

The $-\frac{5}{3}$ slopes of the horizontal-velocity/horizontal-wavenumber spectra lead to a prediction of $k^{-2.4}$ for the vertical wavenumber spectrum of the horizontal velocities (VanZandt, 1982). Again, the observations by Endlich et al. (1969) and Dewan et al. (1984) support the universal spectrum interpretation. The two-dimensional turbulence interpretation does not make any predictions about the vertical wavenumber spectrum because the theory has not been developed sufficiently to include realistic vertical variations, which no doubt are important at the mesoscale. Charney (1971) formulated the theory for synoptic-scale turbulence using the quasi-geostrophic approximations. He found that conservation of potential vorticity leads to a constraint on the relationship between the horizontal and vertical scales that forces the large-scale motions to behave as if they are part of a two-dimensional turbulent process, although vertical variations are important and the vertical gradients are not negligible. No similar treatment has been carried out for the dynamics of mesoscale turbulence, although Lilly (1983) has addressed some aspects of this problem.

It appears that the atmospheric horizontal wavenumber spectra are reasonably universal in the sense that measurements from widely removed locations made under quite different background conditions lead to nearly identical spectra. Not only are the slopes fairly consistent from one observation to the next, but the amplitudes are quite similar. Lilly and Petersen (1983) have shown a composite spectrum derived from several different sources, which gives an idea of the degree of universality. However, so few independent measurements have been made that no firm conclusion can be drawn yet.

In spite of the success of the universal spectrum in explaining some features of the observed spectra, we have to be careful in accepting the universal spectrum concept wholesale from the oceanographers and dismissing two-dimensional turbulence as an unimportant part of mesoscale dynamics. Recently, Olbers (1983), Muller (1984), and Lilly (1984) pointed out that a number of studies of spectra in the oceans indicate the importance of two-dimensional turbulent processes, at least under certain conditions. Thus, while the universal gravity-wave spectra may explain the bulk of the observations, it appears that two-dimensional turbulence predominates at times.

The situation, as it stands, makes it particularly important to obtain observations of spectra of atmospheric variables other than the horizontal velocities. More observations of the spectra of horizontal velocities as a function of horizontal and vertical wavenumber and frequency will not clarify the situation since the slopes that have already been observed are in agreement with both interpretations. Observations of the spectra of the vertical velocity, and other variables such as temperature, are needed to test the universal spectrum predictions, and, if disagreement is found, to provide more data for theoretical developments. Scheffler and Liu (1985) have pointed out that the universal gravity-wave spectrum formulation predicts that the line-of-sight velocity spectra will change slope and power level as a function of zenith angle. They have made a series of calculations of the expected spectral behavior, although the effects of Doppler shifting of the waves by the mean wind was not included. VanZandt (1985) has carried out a similar calculation. Recently, progress has been made in including the Doppler shift effects (Liu, personal communication, 1985). In this paper we will report on observations of line-of-sight velocity spectra made with the Arecibo Observatory 430 MHz radar at zenith angles of 5° to 15°.

2. Description of the data set

The Arecibo Observatory radar operates at a frequency of 430 MHz, corresponding to a wavelength of 70 cm. The peak transmitted power is 3 MW and the average power is approximately 100 kW. The antenna is a circular dish with a diameter of 300 m and is embedded in a valley. The combination of acres of antenna and megawatts of power makes it possible to obtain wind profiles up to heights of 25–30 km by scattering from turbulent variations in the refractive index. The Doppler shift of the signal then gives the line-of-sight velocity of the turbulent scatterers. If the turbulence is moving with the mean flow, the velocity measured with the radar is the same as the velocity of the atmospheric volume sampled by the radar. The technique has been described in reviews by a number of authors including Gage and Balsley (1978), Balsley and Gage (1980), Röttger (1980a), Larsen and Röttger (1982), and Balsley and Gage (1982). The wind measurement technique is also identically the same as what has become known as the wind profiling technique being used as part of the PROFS (Prototype Regional Observing and Forecasting System) network in Boulder, Colorado (Strauch et al., 1984). The technique was first shown to have practical applications by Woodman and Guillen (1974) using the Jicamarca radar located in the Andes Mountains in Peru.

A complete vector wind can be measured by moving the beam into three noncoincident directions. Then all three wind components can be determined unambiguously, subject to errors introduced by variability over the spatial separation between the beams. The Arecibo antenna system was constructed with a feed system that is very massive, appropriate to the scale of the overall system, and that is steered mechanically. Therefore, rapid movement of the antenna is not possible, and five or more minutes of observation time are lost when the look direction is changed by 90° azimuth. The observations presented here are generally for a single azimuth on a given day.

Beginning in 1979, an extensive data set of more than 300 h of wind measurements was gathered with the immediate goal of studying turbulence in the troposphere and lower stratosphere. The data set is summarized in Table 1. Results related to the microscale turbulence characteristics have already been described by Sato and Woodman (1982a,b) and Woodman and Rastogi (1984). Much of the data set consists of consecutive wind profiles at 1-min intervals over periods of 1 h or less, but a few observations of 2 h or longer duration are also available. The length of the data sets is determined by the difficulty of scheduling time at Arecibo in competition with ionospheric physics and astronomy projects. In most of the observations, a fixed-look direction of 10° off-vertical was used, and the azimuth was either toward the east or north. In the original analysis, it was assumed that the line-of-sight component was primarily due to the horizontal wind component, but the data suggests that this is true only for characteristic time scales of approximately an hour or longer. We will return to that point in section 3.

TABLE 1. Wind measurements.

Date	Total observation time (min)	Longest continuous observation (min)	Zenith angle (deg)
24 Feb 1979	305	34	15
25 Jun 1979	418	27	15
26 Jun 1979	85	21	15
17 Jul 1979	346	171	15
29 Aug 1979	185	173	9
30 Aug 1979	126	78	9
10 Oct 1979	252	25	15
11 Oct 1979	143	29	15
12 Oct 1979	512	36	15
13 Oct 1979	107	30	15
20 Dec 1979	281	44	15–10–7.5
21 Dec 1979	227	32	7.5
22 Jan 1980	473	380	10
23 Jan 1980	461	298	10
24 Apr 1980	7	7	15
26 Apr 1980	47	8	12–8.4–5.4
27 Apr 1980	8	8	5.4
3 May 1980	42	10	7.5

3. The frequency spectra

The frequency spectra were first calculated for the longest continuous data series available. The first started at 0119 AST (Atlantic standard time) on 22 January 1980 and ended at 0740 AST, covering just over 6 h and 20 min with 380 consecutive profiles. The second extended data set began at 0110 AST on 23 January 1980 and ended at 0608 AST, covering almost 5 h with 298 consecutive profiles. In both cases, the zenith angle was 10° and a pulse width of 1 μs was used, giving a height resolution of 150 m. The beam width of the 430 MHz system is 0.17°, but that is not relevant for the data presented here. Since the near-field extends to a height of close to 250 km, all measurements in the troposphere and stratosphere are within that regime. Thus, the radar beam has to be treated as a searchlight beam with a constant diameter of a little less than 300 m, the diameter of the dish. That also means that no inverse-range-squared correction for the received power is necessary, as would be the case in the far field of the antenna radiation pattern.

The received power for the two periods is shown in Figs. 1a, b as a gray-scale plot. The vertical axis is height above mean sea level and the horizontal axis is time. The darker areas correspond to higher power levels.

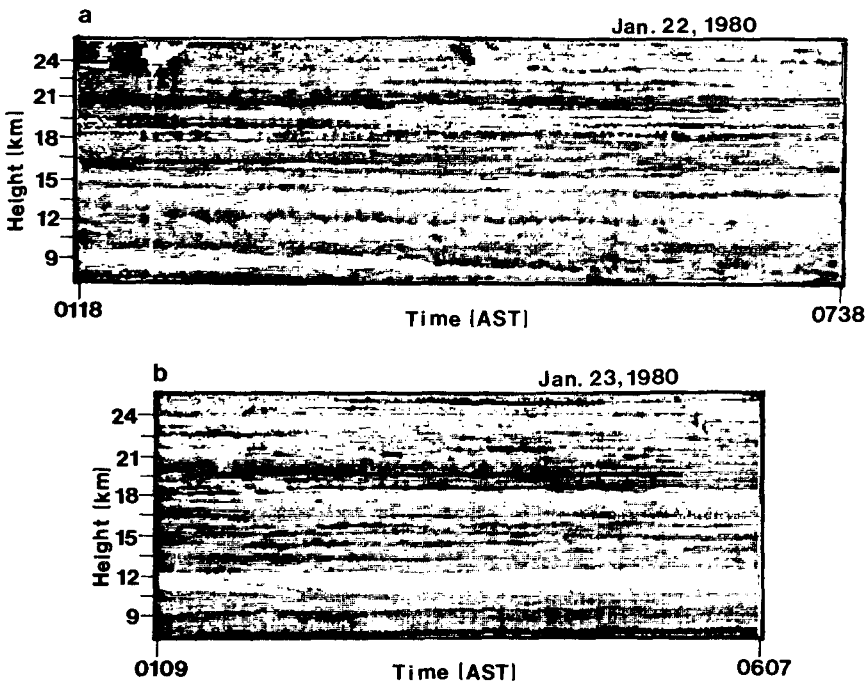


FIG. 1. Radar reflectivity as a function of time and height measured with the Arecibo Observatory 430 MHz radar on (a) 22 January and (b) 23 January 1980. Darker regions indicate higher reflectivities.

The features evident in the two plots are typical of observations in a stable atmosphere. Later we will present similar plots made during periods of convective activity when there is much less horizontal stratification. Sato and Woodman (1982a,b) have discussed the Arecibo observations as they relate to small-scale turbulence. Typical characteristics are the thin turbulent layers evident in the stratosphere. Layers are also present in the troposphere, but they are thicker and, at times, move rapidly downward. The layers in the stratosphere are nearly stationary in height, but there appears to be a small downward displacement with time. The layers may be associated with certain phases of wave structures, as postulated for the thermocline in the oceans. Recently, Fritts and Rastogi (1985) discussed observations and theory related to generation of turbulent layers in the atmosphere by gravity waves.

The spectrum for the velocities obtained on 22 January 1980 is shown in Fig. 2a. The mixed-radix, Fast-Fourier-Transform developed by Singleton (1967) was used to calculate the spectrum for each height. The time series were detrended and a Hanning window covering the first and last 10% of the time series was applied. In any given velocity profile, there will be certain heights where the signal-to-noise ratio is too low to determine the value of the velocity. That will always be the case at high altitudes, and it will also happen at lower heights, usually between turbulent layers. Several schemes were applied to handle the missing data points. First, the missing values were replaced with interpolated values, and the average spectrum for all heights up to 24 km was calculated. Second, only those heights with no more than three missing data points were used to calculate the average spectrum. Finally, only those heights with no missing data points were used to calculate the average spectrum. The result of the latter approach is shown in Fig. 2a. However, there was no substantial difference in the slope of the spectrum or the power level, regardless of which approach was used. Two power density scales are given in Fig. 2a, and in subsequent power spectral density plots. The left-hand scale is the power density for the line-of-sight or radial velocity measurements. The right-hand scale is the power density if the radial velocity is the projection of a purely horizontal velocity on the line-of-sight direction. The left- and right-hand scales are related by a factor of $\sin^2\theta$ where θ is the zenith angle.

The average spectrum for the data taken on 23 January 1980 was calculated in the same way as the spectrum shown in Fig. 2a, and the result is shown in Fig. 2b. There is a broad peak in the spectrum at a period around 30 min. The wave oscillations associated with the peak are easily discernible at heights between 13 and 16 km in the corresponding velocity time series shown in Fig. 3, but the source of the wave structure is not known. The velocities in the figure can be scaled by noting that the separation between tic marks on the vertical axis is equivalent to 1 m s^{-1} in radial velocity. Coherent oscillations such as this are not a common feature in the data.

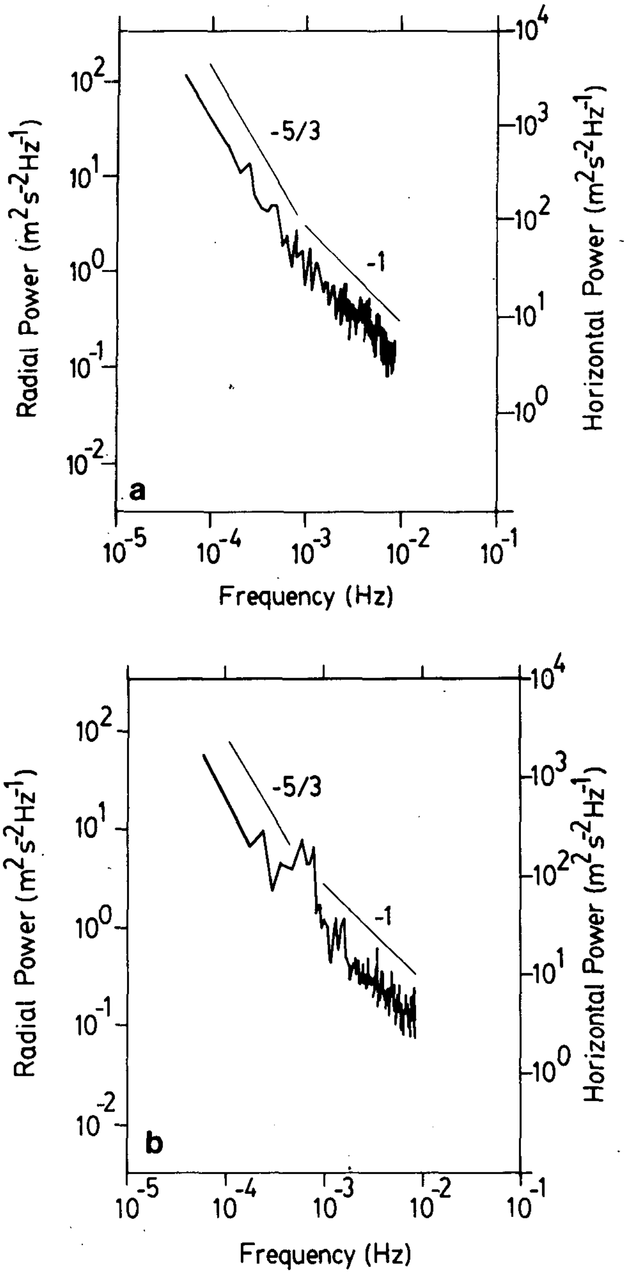


FIG. 2. (a) Average power spectrum as a function of frequency for heights between 7.5 and 21.0 km for measurements made on 22 January 1980. The change in slope from $-\frac{5}{3}$ to -1 near a period of 1 h is attributed to the increasing contribution of the vertical velocity to the line-of-sight velocity at higher frequencies. The spectral index for the entire spectrum calculated from a least-squares fit has a value of -1.14. (b) As in (a) but for data taken on 23 January 1980. The calculated spectral index is -1.20.

For completeness, slopes were fitted to the spectra in Figs. 2a, b by a least-squares method. The slopes were, respectively, $n = -1.14 \pm 0.10$ and $n = -1.20 \pm 0.10$. The error bars were calculated by the method described by Bevington (1969). A slope near $-\frac{5}{3}$ would be expected if the line-of-sight velocities were dominated by the projection of the horizontal velocity in the direction of the radar beam. However, the earlier analysis of Poker Flat data by Balsley and Carter (1982) has already shown that a change of slopes occurs for oblique velocity spectra at periods near 1 h. The slope at lower frequencies is near $-\frac{5}{3}$ and, at higher frequencies, near -1. There appears to be the same change in slope in our spectra as is seen by comparing the spectra in Figs. 2a, b with the $-\frac{5}{3}$ and -1 reference lines plotted in the figures. However, there are only a few points at the low frequency end that define the $-\frac{5}{3}$ slope. One reviewer pointed out that simple gravity wave theory predicts equal vertical and horizontal velocity contributions at a period close to 30 min; the intersection of the $-\frac{5}{3}$ and -1 reference lines is at a period of 20–25 min which is in good agreement. The Brunt–Väisälä periods calculated from the San Juan radiosonde data for 1200 GMT on 22 January 1980 were in the range from 3.5 to 9 min for the height range used to calculate the spectra in Figs. 2a, b. The longer Brunt–Väisälä periods occurred in the troposphere.

Balsley and Carter (1982) concluded that the change in slope of the frequency power spectra at periods of less than an hour was a result of the vertical velocity contribution, which becomes increasingly important at higher frequencies. They analyzed a short data set obtained with a single vertically pointing beam and found a slope of -1 for the vertical-velocity frequency spectra, in support of their interpretation of the oblique spectra. We can test this idea further by using Arecibo observations made at different zenith angles. The spectrum at the smaller zenith angle will then have more power than the one at the larger zenith angle if we are measuring the vertical velocity. Unfortunately, there are only a few examples of such observations in the Arecibo data set. However, on 20 December 1979, data were taken over a half-hour period at 15° zenith angle, and within 6 h a second set of observations was made at 7.5° zenith angle. Data were taken at 10° zenith angle, as well, but the time series was too short to be useful. The resulting spectra for 7.5° and 15° are shown in Fig. 4, where the dashed curve corresponds to the smaller zenith angle. If the line-of-sight velocities were due to the projection along the beam of a purely horizontal wind vector, then the ratio of the power spectral densities at 15° and 7.5° zenith angle should differ by a factor of $[\sin(15°)/\sin(7.5°)]^2$ or approximately 4. The spectra should be nearly coincident, as is the case, if the true velocity is vertical. The ratio of the power levels would be $[\cos(15°)/\cos(7.5°)]$ or 0.95 if the true velocities are vertical. A cautionary note: the two data sequences that make up the spectra in Fig. 4 were taken 6 h apart; changes in the spectral power level over that period would confuse the analysis, although other spectra in the data set measured over similar time intervals at a constant zenith angle in nonconvective conditions do not exhibit any substantial variations in

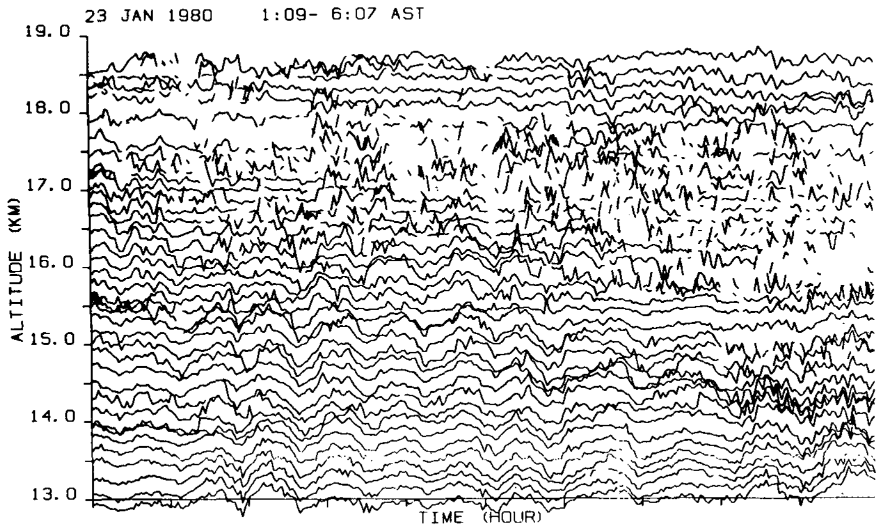


FIG. 3. A sample of the velocity time series at heights between 13.0 and 18.5 km on 23 January 1980. Note the strong wave oscillations with a period between 30 and 50 min evident in the height range from 13 to 16 km. The spacing between tic marks on the vertical axis corresponds to 1 m s^{-1} radial velocity.

power levels. The spectra in Figs. 2a, b are good examples. The time series are separated by more than 17 h but any difference in the power levels is indistinguishable.

Care is needed in calculating spectra for comparison with theory. The effects of windowing due to a finite length time series and the effects of trends have been discussed in various texts (e.g., Bath, 1974). However, the slope of oblique velocity spectra can also be affected by averaging, something that is often done to decrease the errors in the spectral estimate. To show this effect, the velocities were averaged over height before the spectrum was calculated, instead of calculating the spectrum for each height first and then averaging the spectra. Figure 5 shows the spectral index as a function of height for (a) the spectrum of the average time series for ten adjacent heights (solid line) and (b) the average spectral index for the spectra at ten adjacent heights (dashed line). The spectrum is steeper at all heights for

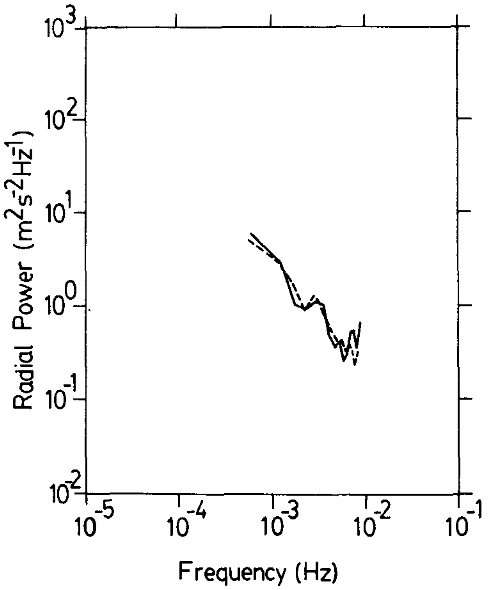


FIG. 4. Frequency power spectra measured at zenith angles of 7.5° (dashed line) and 15° (solid line) on 20 December 1979. The spectra should differ by a factor of approximately 4 in power level if the horizontal velocity contribution was dominant.

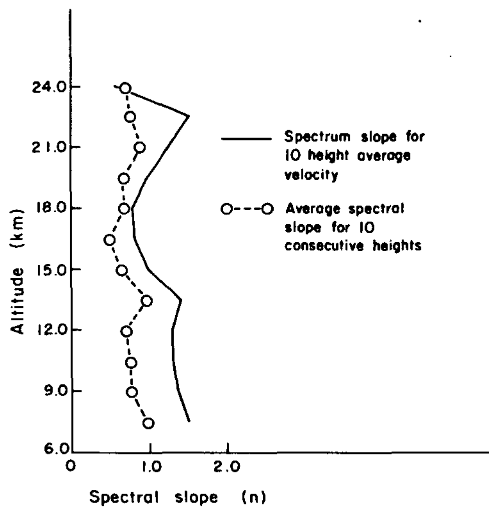


FIG. 5. Spectral index as a function of height. The dashed line shows the spectral slope when the spectra were calculated from the average velocity over ten consecutive heights. The solid line shows the slope when the spectra at ten consecutive heights were averaged.

the averaged velocities. When the velocities for ten consecutive heights were averaged and the spectrum calculated, the average spectrum for the entire height range had a slope of −1.30, as opposed to a value close to −1 when the spectra were calculated separately for each height. We expect that radars with poor height resolution will produce steeper oblique spectra since the radar effectively averages the velocities over the height range of a range gate, but weighs the velocities by the distribution of the signal strength within the gate. We will return to the discussion of the steeper slope produced by averaging in the time domain in section 4.

Scheffler and Liu (1985) pointed out that the line-of-sight, one-dimensional spectrum measured by a radar wind profiler will have contributions from both the vertical and horizontal velocities for off-vertical zenith angles. They assumed that the universal gravity-wave spectrum is the appropriate model and calculated the frequency spectrum and line-of-sight wavenumber spectrum as a function of zenith angle. The gravity wave polarization relationships provide the link between the wave-associated vertical and horizontal velocities. Their results for frequency spectra are summarized in their Fig. 1, which shows a −2 slope for the spectra at periods greater than 80 min, except at zenith angles very close to vertical when the slope approaches zero, as expected from the Garrett–Munk (1975) formulation. The slope decreases to a value close to zero at frequencies of 10^{-3} Hz and greater. The spectra presented here are in good agreement at frequencies less than 3×10^{-4} Hz but do not display the shallow, near-zero slope at the higher frequencies. Scheffler and Liu's (1985) calculations also predict a difference in the power level of the line-of-sight spectra of approximately one-half decade when the zenith angle is increased from 7.5° to 15°. The difference in the level of the radial power spectra at those two zenith angles, as we discussed earlier, is much less. Scheffler and Liu (1985) analyzed the frequency spectra calculated from velocity time series measured in March 1981 with the Arecibo Observatory 430 MHz radar and found a spectral slope of −2 for frequencies less than 2×10^{-3} Hz. Examination of their Fig. 4a also shows a shallower slope near −1 for higher frequencies. Their observations were taken at a zenith angle of 10°, similar to a number of the observations presented here. Thus, there is rough agreement between their spectra and our spectra, but the change in slope in their spectra occurs at a higher frequency than in our study and that of Balsley and Carter (1982).

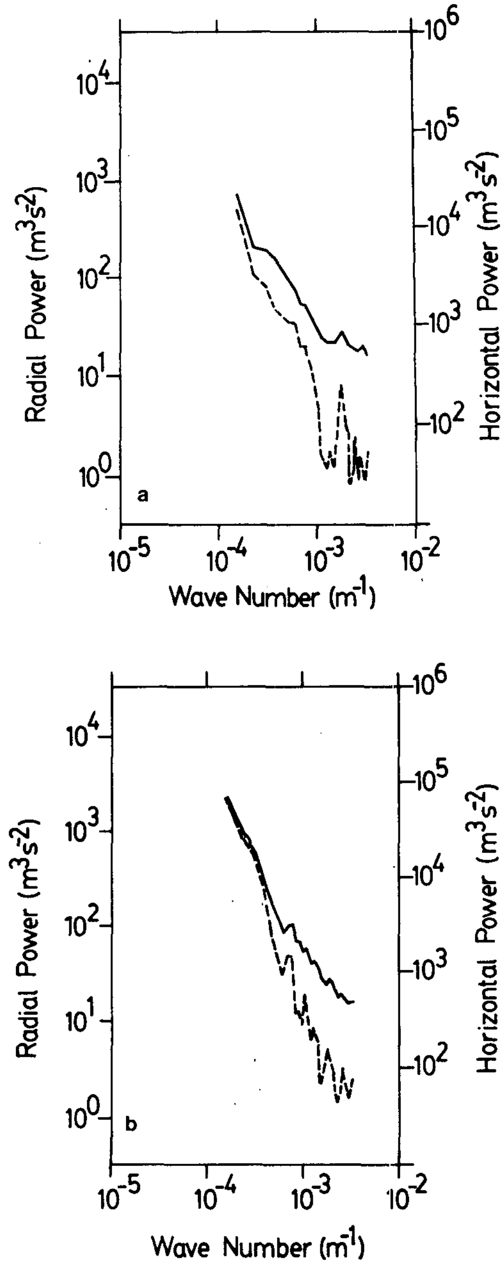


FIG. 6. (a) Average vertical wavenumber power spectrum over the height range from 7.5 to 21.0 km measured on 22 January 1980. The solid line shows the average of 380 power spectra calculated from consecutive velocity profiles. The dashed line shows the spectrum calculated from the average of 380 consecutive wind profiles. As in (a) but for measurements made on 23 January 1980. This spectrum is based on 298 wind profiles.

4. Radial wavenumber spectra

The excellent height resolution of the measurements makes it possible to calculate the vertical wavenumber spectra of the velocity profiles. In Fig. 6a we show the result of calculating the vertical wavenumber spectrum for each of the 380 profiles obtained on 22 January 1980 and averaging those spectra as the solid line. The least-squares fit of the slope gives a value of −1.17 ± 0.08. This value corresponds to the radial wavenumber spectrum of the oblique velocity and is considerably shallower than the vertical wavenumber spectra of the horizontal velocity measured in earlier studies by

Endlich et al. (1969) and Dewan et al. (1984). In those studies, spectral slopes near −2.5 were found.

The same calculation was made for the first 298 profiles obtained on 23 January 1980. The spectrum is shown as the solid line in Fig. 6b. The least-squares slope had a value of -1.46 ± 0.06, steeper than on the previous day, but considerably more shallow than the value of −2.5 expected for the horizontal velocity.

We averaged all of the velocity profiles, 380 and 298, respectively, for the two time series, and calculated the radial wavenumber spectrum of the resulting average wind profile. We expect that averaging over time will reduce the vertical velocity contribution. Then the spectra should be steeper, in agreement with the results of Endlich et al. (1969) and Dewan et al. (1984), if the primary contribution is from the horizontal velocity. Figures 6a, b show the spectra of the average profiles as the dashed lines, which have least-squares slopes of -1.88 ± 0.16 and -2.27 ± 0.18, still less than the expected value of −2.5 but approaching the expected value. Scheffler and Liu (1985) also calculated line-of-sight wavenumber spectra from data taken with the Arecibo 430 MHz radar in March 1981. They found a spectral slope close to −2 in their spectra taken at a zenith angle of 10°.

It seems reasonable that averaging in time will reduce the vertical velocity contribution to the line-of-sight velocity measured at oblique zenith angles. We showed in section 3 that averaging in height apparently also reduces the vertical velocity contribution. These two effects should provide a good test for theories proposed to explain the observed mesoscale power spectra since they show the link between the vertical wavenumber and frequency domain.

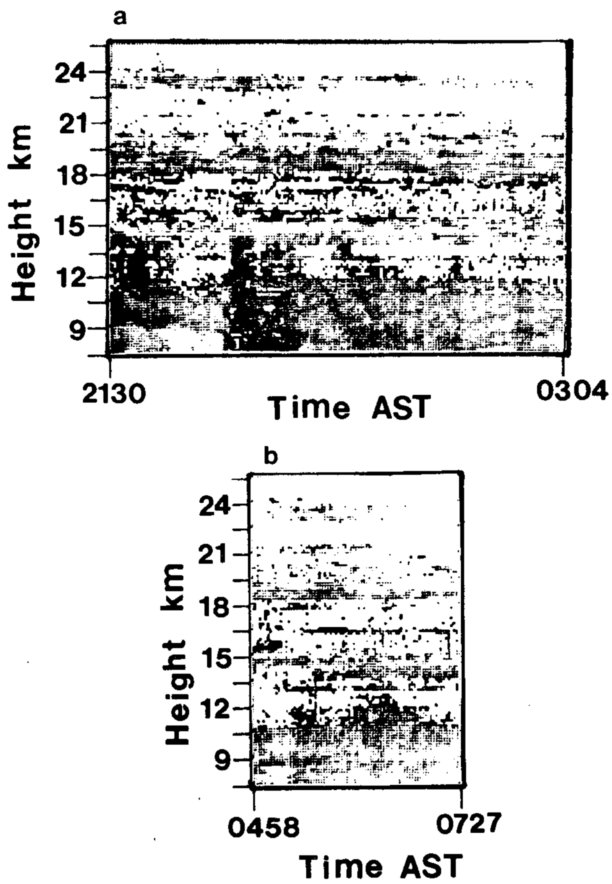


FIG. 7. (a) Reflectivity as a function of height and time on a day with convective activity. (b) Reflectivity later in the day when the convective activity has diminished. Compare this plot to the one shown in Figs. 1a and 1b.

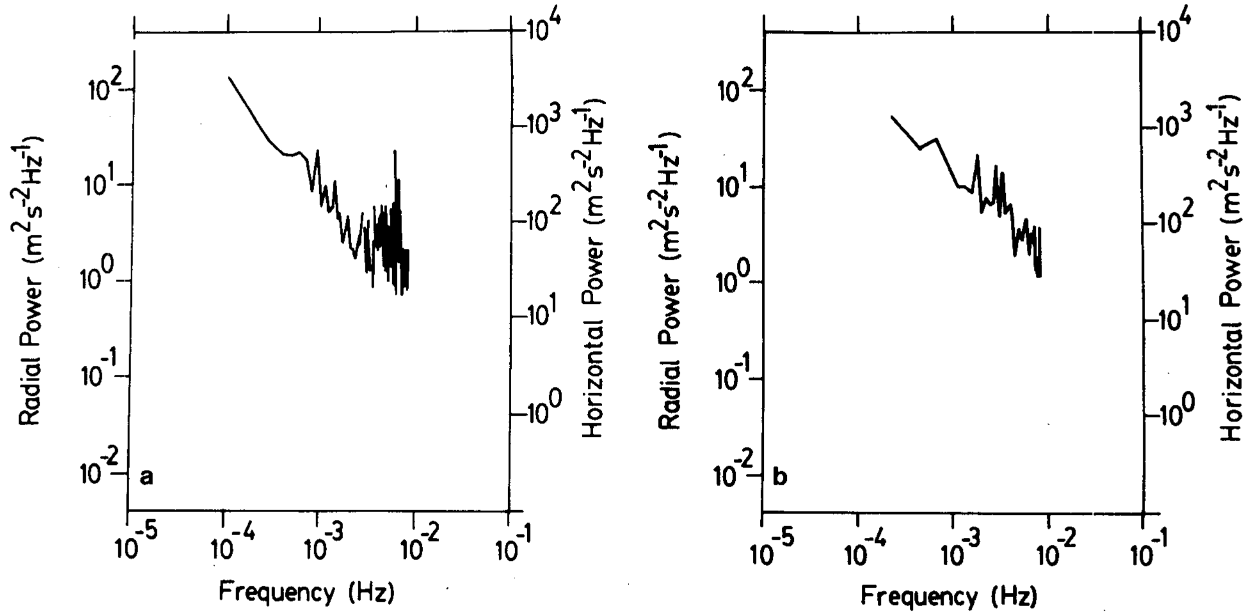
5. Effect of convective activity

On 29 and 30 August 1979, convective activity developed above the Arecibo radar in the late evening and diminished in intensity during the early morning hours of the following day. Continuous data are available for 173 min beginning at 2130 AST on 29 August and again for 78 min beginning at 0458 AST on 30 August. There was a gap of just under 2 h between the end of the first data series and the beginning of the next.

The radar reflectivities are shown in Figs. 7a, b. There is much less stratification of the type evident in Figs. 1a and 1b, obtained during stable conditions. Also, there is considerable vertical structure in the reflectivities associated with the convection. The reflectivities for the second period show a more stratified and more stable environment than in the first period, although still not as stable as the environment shown in Fig. 1.

Figure 8a shows the frequency spectrum calculated for the velocities measured during the first period. A slope close to −1 is evident for the spectrum as a whole, but there is a strong peak in the power spectral density at a period of 6–7 min. The peak is close to the Brunt–Väisälä period calculated from the San Juan radiosonde ascent at 0000 GMT on 30 August in the upper troposphere and presumably is due to the local convective activity. Figure 8b shows the frequency spectrum for the second period, less than 2 h after the end of the first period. The peak at the Brunt–Väisälä period had disappeared at this time. Both spectra show a power level that is five–six times greater than the spectra obtained during stable conditions such as those shown in Fig. 2.

Figure 8c shows the two spectra superimposed, with the solid line representing the spectrum during the period of convective activity and the dashed line showing the spectrum during the later quiet period. The power in the peak near the Brunt–Väisälä frequency has been redistributed toward the lower frequency end of the spectrum. In particular, the power at frequencies between 5×10^{-3} and 1×10^{-3} s^{-1} has been enhanced. The enhancement at the center of the band is approximately a factor of 4. We calculated the total energy under the curves in the overlapping parts of the spectra



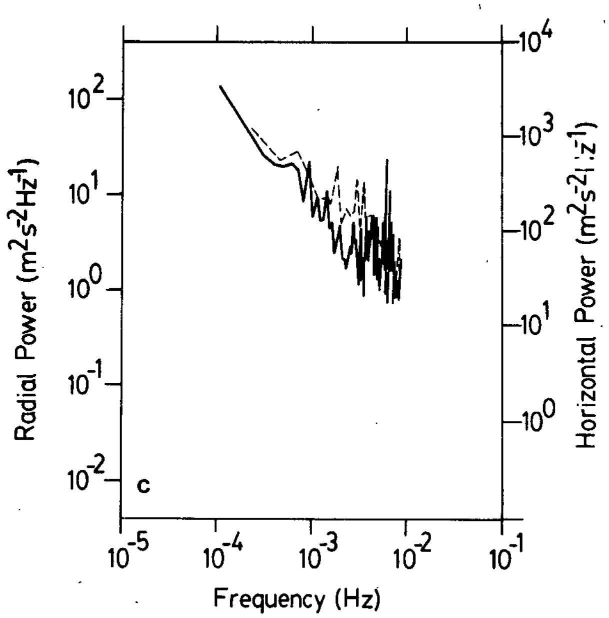


FIG. 8. (a) Average frequency power spectrum at heights from 7.5 to 21 km altitude during the convective activity. Note the strong peak near the Brunt–Väisälä period of 6–7 min. (b) Frequency power spectrum for the period after the convective activity has diminished. (c) The spectra presented in Figs. 8a, b are superimposed for comparison. Note that the later spectrum has more power at lower frequencies. The total energy under the curves in the overlapping frequency range is the same within 10%.

and found that the earlier spectrum had 10% more energy than the later one. Thus, the spectra are consistent with a dynamical process in which energy is essentially conserved but redistributed from shorter to longer time scales and, presumably, from smaller to larger spatial scales.

One interpretation is that the sequence depicted in Fig. 8c is consistent with a two-dimensional turbulent process as described by Lilly (1983). He conjectured that cumulus convection is at the wavenumber boundary between three-dimensional turbulent processes at smaller scales and a two-dimensional turbulence regime at larger scales. Most of the energy associated with convection would pass into the three-dimensional range and energy would propagate toward higher wavenumbers, but a smaller fraction of the energy would cascade toward smaller wavenumbers in the two-dimensional inertial subrange. Our energy calculations show that 90% of the energy is moving toward the low-frequency end of the spectrum, if this is truly a two-dimensional red cascade. The process shown in Fig. 8c is consistent with that scenario, although it cannot be taken as evidence of such a process. It is possible that events conspired to produce an enhancement at lower frequencies and a diminution of the high frequency peak without the energy actually propagating along the spectrum.

The latter could be accomplished through a combination of advection of low frequency energy into and propagation of gravity wave energy out of the volume observed by the radar.

A peak near the Brunt–Väisälä frequency similar to that shown in Fig. 8a was observed on 17 July 1979, when observations were made in a convective environment. At that time no data were obtained during the ensuing stable conditions.

6. Discussion

The role of a universal gravity wave spectrum and two-dimensional turbulence in mesoscale dynamical processes is still not clear. The results presented here imply that spectra of the vertical velocities show slopes near -1 for the frequency spectrum and between -1.2 and -1.5 for the vertical wavenumber spectrum. Neither slope is in agreement with the predictions of the universal gravity-wave spectrum as described by VanZandt (1982). The slope of the horizontal velocity frequency spectrum has been established to be close to $-\frac{5}{3}$. It then follows from the universal spectrum formulation that the slope of the vertical velocity frequency spectrum should be $+\frac{1}{3}$. Also, the vertical wavenumber spectrum for the vertical velocity should have the same slope as for the horizontal velocity spectrum, namely -2.5.

Although our inferred frequency spectra for the vertical velocities show slopes that are steeper than those expected from the predictions of the universal gravity-wave theory, observations by Röttger (1980b) using the SOUSY-VHF-Radar located in the Harz Mountains and by Ecklund et al. (1985) using data from a radar network located in the Rhone delta have shown flat vertical velocity spectra with a peak near the Brunt–Väisälä frequency on occasion. The study by Ecklund et al. (1985) has shown that a slope close to zero occurs under low background wind conditions, but the slope steepens to a value close to $-\frac{5}{3}$ when the background wind speeds are higher. The Arecibo frequency spectra calculated from the data sets listed in Table 1 show a slope close to -1, and the examples presented in this article are representative of the data set as a whole.

The theory of two-dimensional turbulence in the mesoscale range has not been developed sufficiently to account for realistic variations in the vertical direction and to account for vertical velocities. Lilly (1983) treated the relevant equations and showed that waves and turbulence could coexist in the range of interest, but predictions of the spectral slopes for quantities other than the horizontal velocity are not available. Charney (1971) showed that a quasi-geostrophic flow behaves as two-dimensional turbulence, even when realistic vertical variations are taken into account, due to the constraint imposed by conservation of potential vorticity. Thus, the flow is not inherently two-dimensional but behaves as such. A similar treatment is needed for comparison with mesoscale power spectra.

7. Conclusion

The spectral analysis presented here has led to the inference that the spectra of the vertical velocities follow a -1 slope as a function of frequency and a slope between -1.2 and -1.5 as a function of vertical wavenumber. The observed spectra do not support the universal gravity-wave predictions, at least to the degree of complexity included in the calculations of Scheffler and Liu (1985). On the other hand, the two-dimensional turbulence theory has not been developed sufficiently so that theory and observations of vertical velocity power spectra can be compared. However, the observations may provide a good test of the predictions of any further theoretical developments. Our results have also shown that care must be taken in averaging spectra and velocities in order to decrease the noise in the spectrum since the effect can be to change the spectral slope. The steeper slope obtained by averaging in time or in the vertical direction is evidence of the link between the vertical wavenumber and frequency domain. Finally, we have observed a process that is suggestive of, though not proof of, the existence of a process such as the one described by Lilly (1983) in which energy associated with convection leaks into a two-dimensional inertial subrange on the low wavenumber side of the energy input.

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