AN EFFECTIVE ORBIT ESTIMATION ALGORITHM FOR A SPACE DEBRIS RADAR USING THE QUASI-PERIODICITY OF THE EVALUATION FUNCTION

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ABSTRACT

Orbit estimation of space debris, which is unnecessary objects orbiting around the earth, is an important task in avoiding the collision with spacecrafts. Kamisaibara Space Guard Center radar system was built in 2004 as the first radar facility devoted to the observation of space debris in Japan. In order to detect smaller debris, it is effective to improve SNR (Signal-to-Noise Ratio) using coherent integration. However it is difficult to apply the coherent integration to the real data because the motion of the target is unknown at the first step. We propose fast algorithms for signal detection and orbit estimation for faint radar echoes from space debris by utilizing the characteristic of the evaluation function. The proposed algorithms improve SNR by 10.62 dB. The error of the range and the Doppler velocity with expected one are 89 m and 21 m/s, respectively.

Key words: space debris, signal detection, orbit estimation, detection sensitivity, KSGC.

1. INTRODUCTION

Space debris, or fragments of satellites and rocket bodies orbiting around the earth, cause a space environment problem. More than 2,000 t of artificial objects are in the earth's orbit, and 95 % of them are not functioning. The average velocity of space debris is about 8 km/sec, and it reaches about 10 km/sec in a collision. As the kinetic energy is proportional to the square of velocity, the collision energy is enormous, and thus even tiny debris may cause serious damage to spacecraft.

The number of space debris whose diameter is larger than 1 mm is more than 3.5 million[1][2]. There is the possibility that space debris could collide with operational space craft. For these reasons, it is indispensable to obtain precise orbital information by observing space debris for avoiding accidents. In United States, space debris of about 10 cm size have been observed and cataloged by the existing network of radars and optical sensors around the world by the US SPACECOM[3].

Kamisaibara Space Guard Center (KSGC) radar system was built by the Japan Space Forum as the first radar facility devoted to space debris observation in Japan, and started operation in 2004. Tab. 1 shows main parameters of the KSGC. The KSGC radar is equipped with an active phased array antenna mounted on a rotationary base. It is thus possible to track unknown targets for the full angular region above 15 degree elevation. The orbit can be determined by a single observation pass.

Considering the danger of the collision with space debris, the size region of space debris which we need to observe is more than 1 cm, because it is possible to protect space craft against small debris of less than 1 cm. Currently the KSGC radar system can observe 1 m^2 size of space debris for range 600 km. In the future, we need to improve the detection sensitivity to observe and catalog 1 cm size of space debris. Here we propose a fast algorithm of signal detection and orbit estimation for the faint echoes from space debris.

2. CONVENTIONAL ESTIMATION METHOD USING A SINGLE PULSE

The liner-chirp pulse compression is used for the KSGC radar system to enhance the range resolution and detect distant targets. We define the transmitted signal as $s(t)$, and the received signal after synchronous detection as $r(t)$. They are expressed as

$$s(t) = A(t)\exp(-\mathrm{j}2\pi\frac{B}{2T}t^2), \quad (1)$$

$$r(t) = s(t-t_\mathrm{d})\exp\left[-\mathrm{j}2\pi\{f_\mathrm{d}(t-t_\mathrm{d}) - f_\mathrm{c}t_\mathrm{d}\}\right], \quad (2)$$

Table 1. Main Parameters of the KSGC Radar

Parameters	Value
Location	Kamisaibara, Okayama, Japan (35.31°N, 133.94°E)
Radar System	Active Phased Array
Antenna Size	2.8 m × 2.8 m
Number of Elements	1,395
Gain	38.4 dB
Peak Output	96 kW (69 W × 1,395)
Polarization	Vertical
Center Frequency	3,265 MHz
Beam Width	1.9°
Range Resolution	225 m
Band Width	800 MHz
Pulse Length	200 or 300 μsec

where $A(t)$, T, B, t_{d}, f_{d}, f_{c} are the window function, the pulse length, the frequency bandwidth, the delay time, the Doppler frequency and the center frequency, respectively. We apply to the matched filter whose impulse response is expressed as

$$h'(t) = s^*(-t), \tag{3}$$

and obtain the delay time. At this point, the pulse compression ratio P is expressed by $P = BT$. The range resolution is improved $1/P$ times, and the peak output power is enhanced by factor of P[4].

However, the delay time includes an error when the Doppler frequency has an offset. Fig. 1 shows a model of liner-chirp pulse compression. The black line $s(t)$ and $r(t)$ are the transmitted and received signals, respectively. We convolute $r(t)$ with the matched filter without the effect of the Doppler shift $h'(t)$ and obtain the delay time including the error t_l, which is expressed as $t_l = f_{\mathrm{d}}T/B$. We need to consider the Doppler shift in the matched filter as expressed by $h(t) = h'(t)\mathrm{e}^{-\mathrm{j}\omega_{\mathrm{d}}t}$. This means that there is an ambiguity between the delay time and the frequency.

When f_{d} is true we can obtain the true delay time and the response of the filter is maximum. In a conventional method, we search the Doppler frequency which maximizes the response of the matched filter as

$$\max_{f_{\mathrm{d}},t} \left| \int s(\tau)h(t-\tau)\mathrm{d}\tau \right|^2. \tag{4}$$

Fig. 2 shows an example of the conventional method applied to the real data of the KSGC radar. The left and right figures are echoes from ISS (International Space Station) and H2A-Rocket Booster (H2A-R/B), respectively. Their RCS (Radar Cross Section) are about 388.64 m^2 and 27.1 m^2, respectively. Fig. 2 shows that it is difficult to detect a signal in low SNR condition using the conventional method.

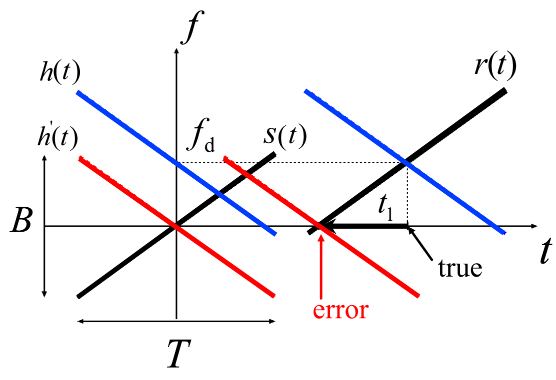

Figure 1. Liner-chirp Pulse Compression.

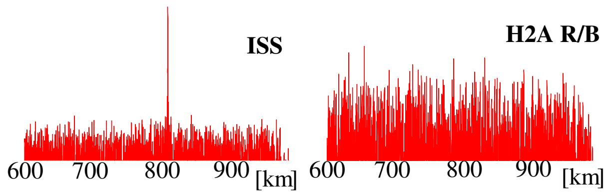

Figure 2. Example of Conventional Method Applied to the Real Data.

3. EVALUATION FUNCTION WITH A COHERENT INTEGRATION

In order to improve SNR, we try to integrate received signals coherently. However, coherent integration is difficult in the space debris observation because the range, the Doppler velocity and the direction of the movement of debris are unknown. Therefore, we need to assume a motion model.

The instantaneous orbit of space debris is an ellipse with one of its focus at the center of the gravity of the earth. However, it can be approximated as uniform motion for a duration of few seconds as the result of our examination. We approximate the orbit of space debris as a straight line in the vicinity of the radar, and take the coordinate plane to include the orbit and the radar antenna.

Fig. 3 shows the system model. We can express the orbit with 3 parameters $(r_1, v_{\mathrm{d}}, \phi) = \boldsymbol{x}$, and the range of debris is expressed as

$$r_{\mathrm{d}}(t) = \sqrt{r_1^2 - 2r_1 v_{\mathrm{d}} t + \left(\frac{v_{\mathrm{d}}}{\cos\phi}\right)^2 t^2}, \tag{5}$$

where r_1, v_{d}, and ϕ are the range, the Doppler velocity, and the angle between the direction of the motion and that of the line-of-sight of the first echo, respectively. We treat the orbit estimation process as an optimization problem. We integrate received signals

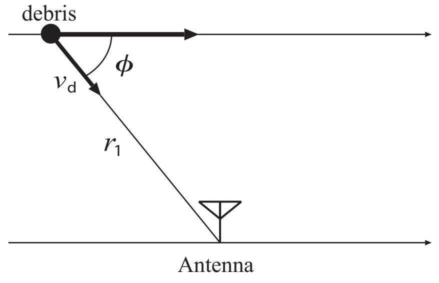

Figure 3. System Model.

coherently according to Eq. 5 and search the optimum orbit parameters which maximize the output as

$$\max_{\boldsymbol{x}} \left| \sum_{i=1}^{N} \int s_i(\tau) h_i(t_i - \tau) \mathrm{d}\tau \cdot \mathrm{e}^{-\mathrm{j}(\omega_i t_i - \theta_i)} \right|^2, \quad (6)$$

where $s_i(t)$ is the signal for i-th pulse, $h_i(t)$ is the impulse response with a Doppler shift, N is the number of integration, and $t_i, \omega_i, \theta_i (i = 1, \cdots, N)$ are uniquely determined by $\boldsymbol{x}$. The parameters t_i, ω_i, and θ_i are the delay time revision, the frequency revision and the phase revision between adjacent pulses, respectively.

The transmitted signal is a liner-chirp signal. Fig. 4 shows a part of the evaluation function for the true ϕ, where $\boldsymbol{x}$=(800 km, 4 km/s, 60°), $N = 4$, IPP (Inter Pulse Period) of 7,500 μsec, and pulse length $T = 300$ μsec. There are two noticeable points in this evaluation function. First, we can see the correlation between r_1 and v_d. As we explained in Sec. 2, it is the ambiguity caused by the liner-chirp pulse compression and its gradient is $a = B/f_\mathrm{c}T$. Secondary, we see the periodical peaks in the direction of v_d. We define this period as v_d0. This is because even if the phase rotates between adjacent pulses by the integral multiples of the wavenumber, we can integrate received signals coherently. Therefore, many suboptimal solutions are generated and this period is expressed as

$$v_\mathrm{d0} \simeq \lambda / 2T_\mathrm{IPP}, \quad (7)$$

where λ and T_IPP are wavelength and IPP, respectively. As $\lambda = 9.18$ cm and the region of T_IPP is 5–10 msec in case of KSGC, we need to search for v_d in the accuracy of mm/s, which is not realistic. Therefore an effective searching method is required. Here we divide the method to optimize this evaluation function into two stages; the signal detection and the orbit estimation.

4. FAST SIGNAL DETECTION ALGORITHM

Here, we propose the signal detection method. Its purpose of the algorithm is to detect desired echo

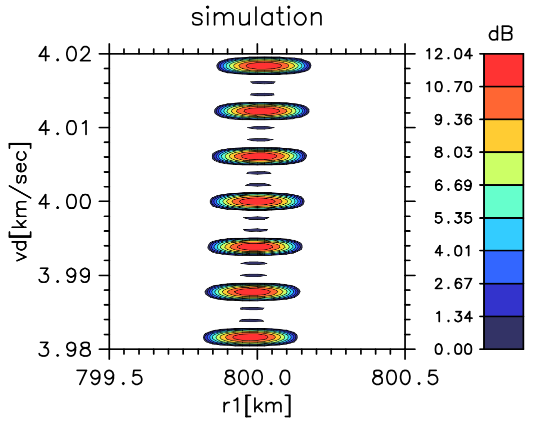

Figure 4. Evaluation Function.

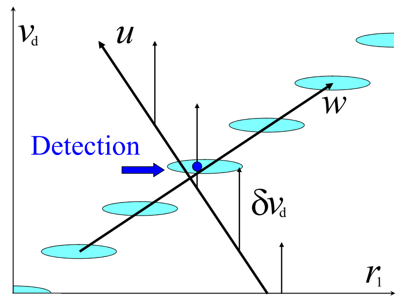

Figure 5. Model of the Detection Method.

from the target even if orbital parameters cannot be estimated. It means that we need the algorithm which can efficiently search for any suboptimal solutions.

We found out that the effect of v_d is predominant in θ_i as the result of our examination of this evaluation function. It shows that tuning v_d can revise all phase shifts of pulses which are used by coherent integration. We employ two approaches: First, we use the correlation between r_1 and v_d. Fig. 5 shows the model of the proposed detection method. We define the orthogonal axis and the parallel axis to the direction of the correlation as the u axis and the w axis, respectively, as shown in Fig. 5. The axis of the correlation can be found by searching along the u axis.

Secondary, we introduce δv_d which tunes all phase shifts of pulses. As we explained in Sec. 3, θ_i is uniquely determined by the orbital parameters $\boldsymbol{x}$. According to Eq. 7, we divide the Doppler velocity in θ_i into nv_d0 and δv_d. They are expressed as

$$v_\mathrm{d} = nv_\mathrm{d0} + \delta v_\mathrm{d}, \quad (8)$$

where n is an integer and $|\delta v_\mathrm{d}| \leq v_\mathrm{d0}/2$. We define nv_d0 and δv_d as the global Doppler velocity and the local Doppler velocity, respectively. The global Doppler velocity determines the orbit of debris and influences the response of the filter. The lo-

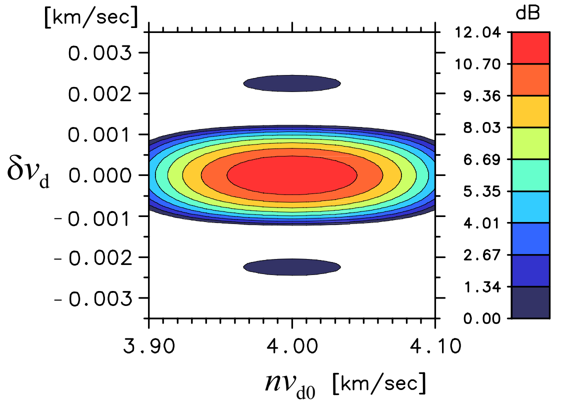

Figure 6. nv_{d0} *vs.* δv_{d}.

cal Doppler velocity δv_{d} determines all phase shifts between adjacent pulses. Fig. 6 shows nv_{d0} vs. δv_{d} in a section of r_1. The evaluation function becomes a smooth and continuous surface by dividing the Doppler velocity, which is one dimension originally, into two dimensions.

In Fig. 5, all parameters are updated when we update u for one step. We search δv_{d} for a period, fixing another parameters instead of simply searching v_{d}. Therefore, one of the suboptimal solutions can be found by searching along the u axis and the δv_{d} axis.

The signal detection algorithm is described as follows:

1. Update u for one step.
2. Update δv_{d}. If $|\delta v_{\mathrm{d}}| < v_{\mathrm{d0}}/2$ take a step towards 3. Otherwise take a step towards 1.
3. If the threshold $<$ the output, take a step towards 4. Otherwise take a step towards 2.
4. Optimize parameters.

We define the time to calculate an evaluation value as k. The calculation time to search in the v_{d} axis for i point is ki. It becomes almost 0 by using the proposed method, since the searching in the v_{d} section can be treated as a simple phase rotation. Therefore, the improvement factor of the calculation time is i. We define the Doppler velocity resolution as Δv_{d}. We need to search in the v_{d} axis for a period by the width of $\Delta v_{\mathrm{d}}/2$. The number of searches is expressed as $i = 2v_{\mathrm{d0}}/\Delta v_{\mathrm{d}}$. In this situation Δv_{d} and v_{d0} are 0.1695 m/s and 6.1213 m/s, respectively, where $N = 16$ and $T_{\mathrm{IPP}} = 7{,}500$ μsec. Therefore i becomes 72 and the calculation time becomes 72 times faster than the v_{d} updating method.

As we explained in Sec.1, the minimum detection size of the KSGC radar system is 1 m^2 for range 600 km. We add a noise to the real data to clarify the improvement of the proposed detection method. From the expected value, we roughly know that the range of H2A-R/B is 895 km. According to the radar equation, the received power from the target whose RCS = 1 m^2 for range 600 km and the target whose RCS = 4.95 m^2 for range 895 km are equivalent. As the RCS of H2A-R/B is 27.1 m^2, its received power is 5.47 times larger than that from the minimum detection size. We add Gaussian noise to the real data to deteriorate the SNR to the condition of the detection limit.

Fig. 7 shows an example of the proposed detection method applied to the experimental data of H2A-R/B, where $N = 16$. The noise level is set to 0 dB. The filter used in the conventional method assumes the correct Doppler velocity, which gives the best result. The left figure shows signals in the region of the observed range. The right figure is a close-up. We can see the peak SNR is improved by 10.45 dB compared with the conventional method. Moreover, the estimated range which is obtained using the conventional method is obviously wrong. On the other hand, the estimated range and the Doppler velocity which are obtained using the proposed method are not improved. However, the estimated parameters still contain substantial errors. Therefore a global optimization is required from these parameters. In the next section, we explain the global optimization method from the detection parameters which are obtained using the proposed detection method.

5. EFFECTIVE ORBIT ESTIMATION ALGORITHM

Here we explain the orbit estimation method. We define the detected parameters using the detection method which is explained in Sec.4 as $\boldsymbol{x}_{\mathrm{det}}$. We search the global optimal solution with the initial parameters $\boldsymbol{x}_{\mathrm{det}}$. A simple method is updating the parameters to increase the evaluation value according to v_{d0} in the direction of w. However, this simple method does not work because the period slightly changes due to the nonlinearity of the Doppler velocity, which shows that the evaluation function has quasi-periodicity. Eq. 7 is satisfied accurately when we assume uniform motion in the line-of-sight. The red line of Fig. 8 shows the model of this simple method. Therefore, the obtained parameters by using this simple method are not accurate enough.

In order to obtain parameters accurately, we jump to the next area according to v_{d0} in the direction of w and optimize every suboptimal solutions. The blue and the black lines of Fig. 8 show the model of this method. However, this method requires a long calculation time because it performs filtering for each updating parameters. Therefore, we propose a new orbit estimation algorithm.

Fig. 9 shows the model of the proposed orbit estima-

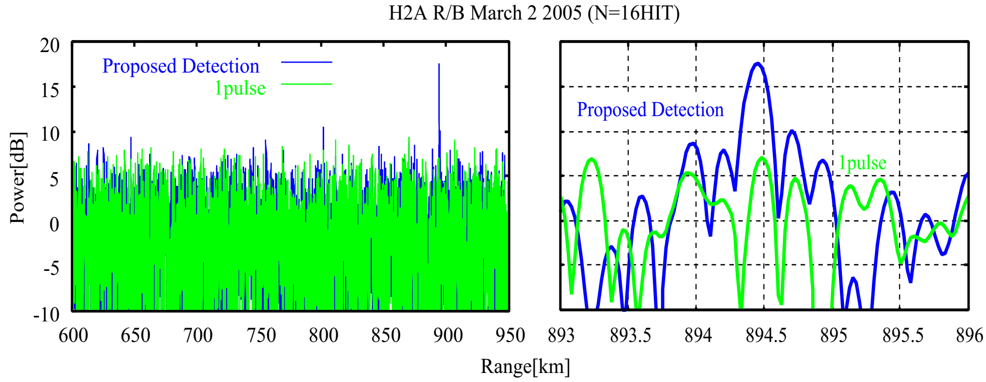

Figure 7. Example of Proposed Detection Method Applied to the Real Data.

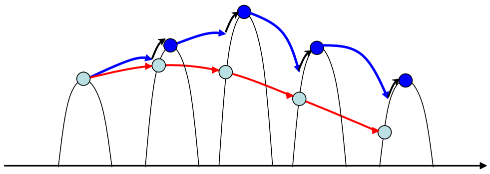

Figure 8. Quasi-period problems.

tion method. We have experimentally clarified that the local Doppler velocity δv_{d} is dominant for the local shape of the evaluation function. In Fig. 9, we regard the local search of v_{d} as the local search of δv_{d}, ignoring the error of the response of the filter and the motion model. Once we optimize r_1 as step 4 in Fig. 9 and find the local peak, we jump to the next area and update all the parameters as step 2 in Fig. 9. The estimated errors of r_1 and v_{d} are small enough compared with the true parameters. Moreover, the calculation time becomes about 25 times faster than the method which performs filtering processing repeatedly, where $N = 16$.

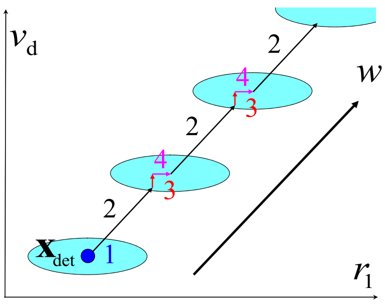

Figure 9. Model of the Estimation Method.

The orbit estimation algorithm is described as follows:

1. Set $\boldsymbol{x}_{\mathrm{det}}$ as the initial parameters.
2. According to v_{d0}, update all parameters in the direction of w.

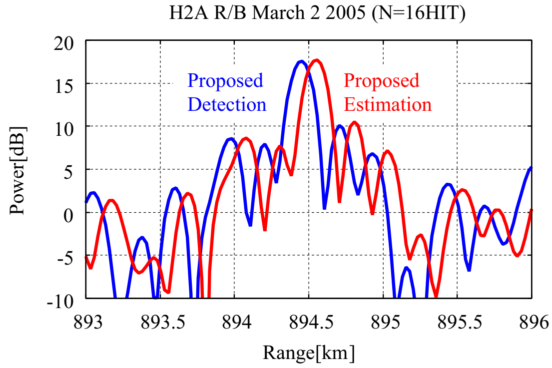

Figure 10. Example of Proposed Orbit Estimation Method Applied to the Real Data.

3. Regard only δv_{d} as a parameter, and optimize δv_{d}.
4. Optimize r_1. Take a step towards 2.

Fig. 10 shows the result of the proposed orbit estimation method applied to the experimental data in Fig. 7. Fig. 11 shows the state of the orbit estimation in the v_{d} section, which gives the evaluation value 0.17 dB higher than the proposed detection method. Tab. 2 is the comparison between the detection, the orbit estimation, and the expected parameters, which shows the error of the estimated parameters are substantially reduced to 89 m for r_1 and 21 m/s for v_{d}, respectively.

Table 2. Comparison of the Detection, the Orbit estimation and the Expected Parameters

	r_1[km]	v_{d}[km/s]	Gain [dB]
Detection	894.450	3.155	17.57
Estimation	894.549	3.235	17.74
Expected	894.460	3.256	

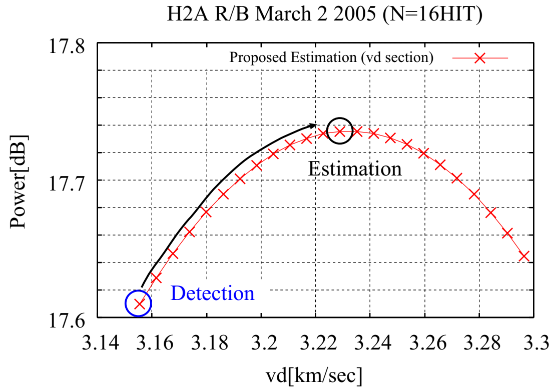

Figure 11. Orbit Estimation (v_d Section).

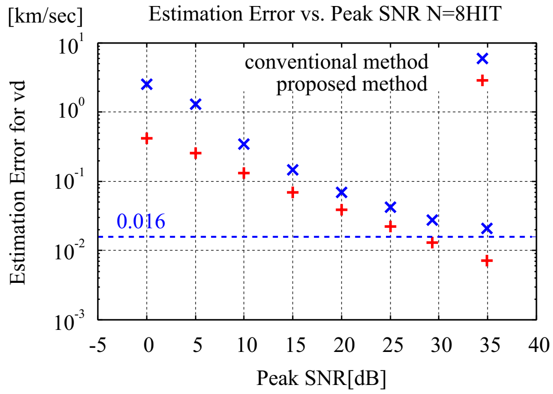

Figure 12. Accuracy Comparison of the Proposed and the Conventional Methods.

6. ACCURACY EVALUATION OF THE ESTIMATED PARAMETERS

We examine the accuracy of the estimated parameters in this section. The conventional method, which we explained in Sec. 2, uses a single pulse. It is unfair to simply compare with the proposed method and the conventional method, since the proposed method uses plural pulses. Therefore, we improve the conventional method to use plural pulses.

We integrate received signals incoherently and search the Doppler velocity which maximize the output as

$$\max_{v_\mathrm{d},t} \sum_{i=1}^{N} \left| \int s_i(\tau) h_i(t - 2iT_\mathrm{IPP} v_\mathrm{d}/c - \tau) \mathrm{d}\tau \right|^2, \quad (9)$$

where c is the velocity of light. We call this method as the conventional method in this section. Eq. 9 shows that we search v_d which maximizes the response of the filter using incoherent integration. This method is simple because it does not require the phases of signals. In this method, however, the estimated error occurs even if SNR is infinity because we assume the orbit of space debris as uniform motion in the line-of-sight. Therefore, the accuracies of the estimated parameters are not enough.

Fig. 12 shows the estimation error vs. peak SNR, where the true $\boldsymbol{x}$=(800 km, 4 km/s, 60°) and $N = 8$. The estimation error is evaluated as RMSE (Root Mean Square Error). In this condition, the accuracy limit of the conventional method is about 0.016 km/s for v_d. This figure shows the accuracy of the proposed method is better than the conventional method for all SNR, particularly in low SNR.

7. SUMMARY

We propose the detection method for the signals from space debris which cannot be detected using a single pulse. We assume the orbit of space debris as uniform motion in this proposed method. The calculation time searching in the v_d section becomes about 72 times faster than the v_d updating method, where the number of the integration $N = 16$ and IPP is 7,500 μsec. The proposed detection method improves SNR by 10.45 dB to the KSGC real data, H2A-R/B, with Gaussian noise to deteriorate the SNR to the detection limit condition.

We propose the method to estimate the orbit of space debris effectively with a high accuracy. The calculation time becomes about 25 times faster than the method which requires filtering processing repeatedly. The proposed orbit estimation method improves SNR only by 0.17 dB compared with the proposed detection method, but the error of estimated parameters are substantially reduced to 89 m for r_1 and 21 m/s for v_d, respectively. The estimation accuracy of the proposed orbit estimation method is better than the conventional method, particularly for low SNR.

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