A Robust and Fast Imaging Algorithm with an Envelope of Circles for UWB Pulse Radars

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Abstract

UWB pulse radar systems are promising as high-resolution imaging techniques for household or rescue robots. We have already proposed a fast imaging algorithm called SEABED based on a reversible transform BST(Boundary Scattering Transform) between the received signals and the target shape. However, the image obtained with SEABED deteriorates in a noisy environment because it utilizes derivative of received data. In this paper, we propose a robust and fast imaging method with an envelope of circles.

1 Introduction

UWB pulse radar systems have a great potential for a high-resolution imaging, which is suitable and efficient for measuring techniques of house-hold and rescue robots. While many imaging algorithm for radar systems have been proposed, they require intensive computations. To solve this problem, we have already proposed a fast imaging algorithm called SEABED (Shape Estimation Algorithm based on BST and Extraction of Directly scattered waves) for UWB pulse radars based on a reversible transform BST between the received signals and the target shape [1, 2]. However, the estimated image with SEABED is not stable in a noisy environment because it utilizes derivatives of the received data. In this paper, we propose a robust and fast imaging algorithm without derivative operation. We utilize circles with the estimated delay for each antenna location. We follow the principle that these circles circumscribe the target boundary [3]. By utilizing this principle and the inverse transform of BST, we prove that the boundary of a convex and a part of concave targets are expressed as a boundary of an union set of these circles. This method does not depend on a derivative of a received data, and enables us to realize a robust imaging even in a noisy environment. We show application examples of the proposed method with a numerical simulation.

2 System Model

We show the system model in Fig. 1. We deal with 2-dimensional problems and TE mode waves. We assume that a target has a uniform permittivity, and surrounded by a clear boundary which is composed of smooth curves concatenated at discrete edges. We also assume that the propagation speed of the radio wave is constant and known. We utilize a mono-static radar system. The induced current at the transmitting antenna is a mono-cycle pulse. We define r-space as the real space, where targets and the antenna are located. We express r-space with the parameters (x, y). An omni-directional antenna is scanned along x axis. Both x and y are normalized by λ, which is the center wavelength of the transmitted pulse. We assume $y > 0$ for simplicity. We define $s'(X, Y)$ as the received electric field at the antenna location $(x, y) = (X, 0)$, where we define Y with time t and speed of the radio wave c as $Y = ct/(2\lambda)$. We apply the matched filter with the transmitted waveform to $s'(X, Y)$. We define $s(X, Y)$ as the output of the filter. We define d-space as the space expressed by (X, Y),

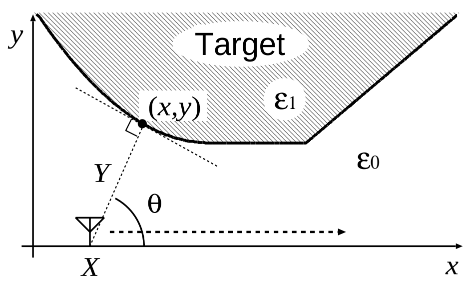

Figure 1: System model.

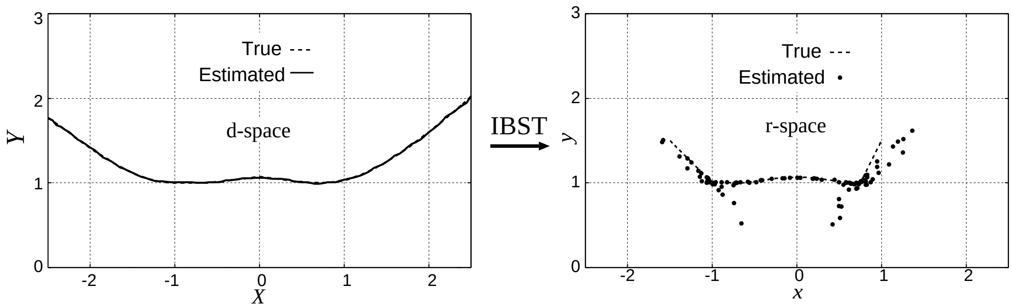

Figure 2: Quasi wavefront with noise (Left side), and an estimated image with the SEABED (Right side).

and call it a quasi wavefront. The transform from d-space to r-space corresponds to imaging which we deal with in this paper.

3 Conventional method

We have already developed a non-parametric shape estimation algorithm called SEABED [1]. This method utilizes a reversible transform BST between the point of r-space (x, y) and the point of d-space (X, Y), which is extracted by the received signal $s(X, Y)$. IBST (Inverse BST) is expressed as

$$\left.\begin{array}{rcl} x &=& X - Y\mathrm{d}Y/\mathrm{d}X. \\ y &=& Y\sqrt{1-(\mathrm{d}Y/\mathrm{d}X)^2}, \end{array}\right\} \qquad (1)$$

where $|\mathrm{d}Y/\mathrm{d}X| \leq 1$ holds. This transform gives us a complete solution for an inverse problem. SEABED has an advantage that it can directly estimate target boundaries with IBST, and achieves a fast and high resolution imaging. However, the estimated image with the SEABED easily deteriorates in a noisy environment because IBST utilizes the derivatives of the quasi wavefront as $\mathrm{d}Y/\mathrm{d}X$. The left side of Fig. 2 shows a quasi wavefront with random error whose standard deviation is 0.01λ. Here, we smooth the quasi wavefront with a Gaussian filter, whose standard deviation is 0.05λ. The right side of Fig. 2 shows the estimated points with IBST. The estimated points with SEABED have large errors, and the maximum error is over 0.5λ, which is not acceptable. We confirm that the estimated point with IBST should exist on the circle whose center is $(X, 0)$ and radius is Y. The angle θ shown in Fig. 1 is determined with $\mathrm{d}Y/\mathrm{d}X$, which is sensitive to a noise. The accuracy of θ strongly depends on that of $\mathrm{d}Y/\mathrm{d}X$. Therefore, the estimated point readily moves around this circle in a noisy environment. To suppress the deterioration of the estimated image with SEABED, the methods for stabilizing images have been proposed [4]. However, they cannot completely remove the error occurred by the derivative operations because they utilize an inverse transform with $\mathrm{d}Y/\mathrm{d}X$.

4 Proposed method

To solve the instability of SEABED, we propose a new imaging algorithm based on an envelope of circles. First, we clarify the relationship between the group of points on a target boundary and the points on an envelope of circles. We assume that the target boundary ∂T is expressed as a single-valued and differentiable function. (X, Y) is a point on the quasi wavefront of ∂T, and we define ∂D as the quasi wavefront. We define Γ as the domain of X for ∂D. We define $g(X, Y) = X - Y\mathrm{d}Y/\mathrm{d}X$, and γ as a domain of $g(X, Y)$. We define $S_{(X,Y)}$ as an open set which is an interior of the circle, which satisfies $(x - X)^2 + y^2 = Y^2$. Fig. 3 shows the relationship between d-space and set of circles in r-space. If ∂D is a single-valued and continuous function, we define S_+ as $S_+ = \{(x, y) \mid (x, y) \in \bigcup_{X\in\Gamma} S_{(X,Y)},\ x \in \gamma\}$. We define ∂S_+ as the boundary points of S_+. Here the next proposition holds

Proposition 1. *If $\partial g(X, Y)/\partial X > 0$ satisfies, the next equation holds*

$$\partial T = \partial S_+. \qquad (2)$$

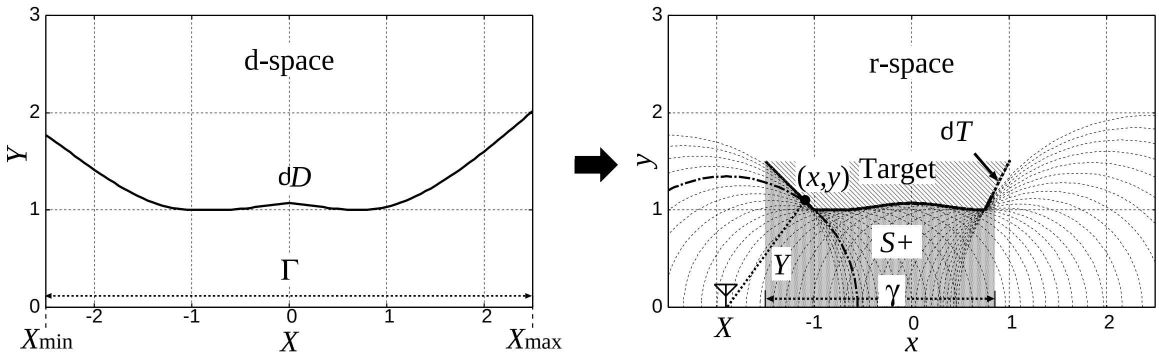

Figure 3: Quasi wavefront in d-space (Left side) and a set of circles in r-space (Right side).

A proof of the proposition 1 is given in the appendix A. Proposition 1 says that ∂S_+ expresses a part of the target boundary. Under the condition $\partial g(X,Y)/\partial X > 0$, all $\partial S_{(X,Y)}$ in $(X,Y) \in \partial D$ circumscribe ∂T. Also $\partial g(X,Y)/\partial X > 0$ satisfies at the case of a general convex target and a part of concave targets as shown in Fig. 3. We also confirm that the edge can be estimated as the intersection point of circles $\partial S_{(X,Y)}$ when a target boundary includes an edge, where (X,Y) is transformed into the edge point (x,y) with the IBST. Therefore, the target boundary ∂T with edges can be expressed as ∂S_+. In our proposed method, we estimate the target boundary with an envelope of circles by utilizing these relationships. This method enables us to transform the group of the points (X,Y) to the group of the points (x,y) without the derivative operation.

We explain the actual procedures of the proposed method as follows. Here we define $R(X,X')$ as x coordinates of the intersection point of $\partial S_{(X,Y)}$ and $\partial S_{(X',Y')}$. We define $X_{\max}$ and $X_{\min}$ as maximum and minimum $X \in \Gamma$. We also define ΔX as the sampling interval of the antenna.

Step 1). Apply the matched filter to the received signals $s'(X,Y)$ and obtain the output $s(X,Y)$.

Step 2). For each X, determine Y as $Y = \max_{Y'} s(X,Y')$, and extracts (X,Y) as quasi wavefront ∂D.

Step 3). Extract boundary points (x,y) on ∂S_+ as $y = \max_{X\in\Gamma} \sqrt{Y^2 - (x-X)^2}$ where x is sampled at an equal interval in the domain Γ.

Step 4). Determine $\partial T = \partial S_+$, $(x_{\min} \le x \le x_{\max})$, where $x_{\min} = R(X_{\min}, X_{\min} + \Delta X)$ and $x_{\max} = R(X_{\max}, X_{\max} - \Delta X)$.

5 Performance Evaluation

We show an application example of SEABED and the proposed method as follows. The signals are received at 101 locations for $-2.5\lambda \le x \le 2.5\lambda$. We add a white noise to the received data $s'(X,Y)$ calculated with the FDTD method. In this case, S/N is about 7.0 dB. The left and right side of Fig. 4 show the estimated image with SEABED and the proposed method, respectively. We set the standard deviation of Gaussian filter as 0.05λ. We confirm that the image obtained with SEABED is not accurate, especially around the edges of the target. On the contrary, the image obtained by the proposed method is stable and expresses an almost accurate target boundary. This is because the proposed method does not spoil the information of the inclination of the target shape. The calculation time of the algorithm is within 0.1 sec with a Xeon 3.2 GHz processor, which is short enough for a realtime imaging.

To evaluate a limitation of the imaging stability v.s. S/N, we introduce two evaluation values, which are mean value of the error defined as $\mu = \frac{1}{N}\sum_{i=0}^{N} |e(x_i)|$, and standard deviation of the errors defined as $\sigma = \sqrt{\frac{1}{N}\sum_{i=0}^{N}(|e(x_i)| - \mu)^2}$ where $e(x) = y_{\rm e}(x) - y_{\rm true}(x)$. Here, $y_{\rm true}(x)$ is the true target boundary, and $y_{\rm e}(x)$ is the estimated image, and N is number of the estimated points. The left and right side of Fig. 5 shows that μ and σ for S/N in the case of the target as shown in Fig. 2. We obtain 5 times improvement for μ, and 2 times improvement for σ compared to those of SEABED when S/N is over 7 dB. These improvements are obtained regardless of the noise power. We should notice that this method achieves a fast and robust imaging, which cannot be obtained with the conventional algorithms. It is important future work to extend this method for a

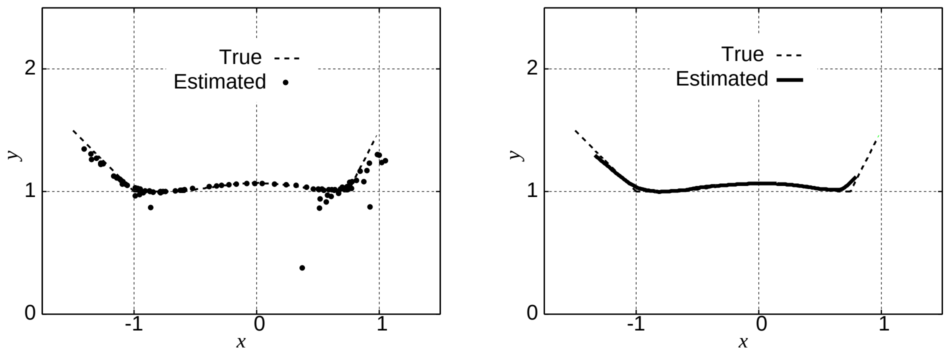

Figure 4: Estimated image with the SEABED (Left side) and the proposed method (Right side).

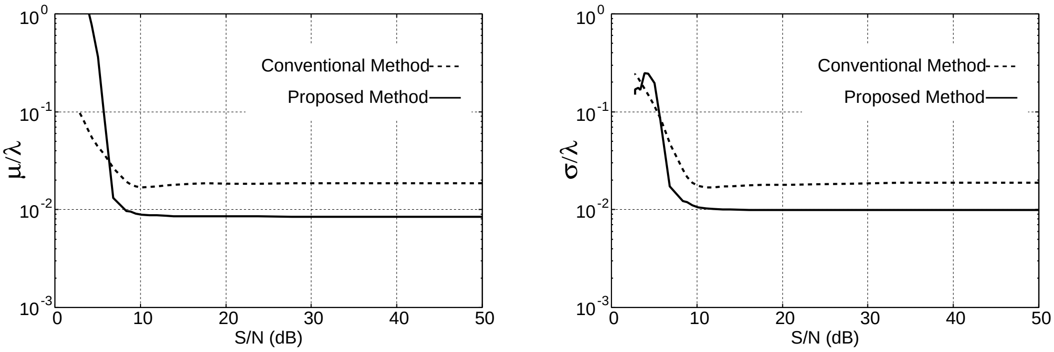

Figure 5: μ (Left side) and σ (Right side) for S/N of an each method.

general target shape including the concave with a large curvature.

6 Conclusion

We proposed a robust and fast imaging method with an envelope of circles. We clarified that a general convex target and a kind of a concave target boundary can be expressed as a boundary of an union set of circles with time delays. We also showed that the application range of the proposed method. In the numerical simulation, we clarified that the proposed method achieves a stable imaging compared with the SEABED. Besides, we confirmed that the proposed method achieves a fast imaging like SEABED algorithm.

Acknowledgment

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References

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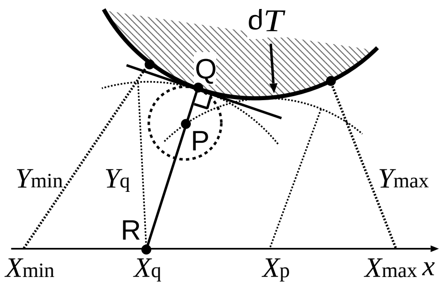

Figure 6: Relationship between ∂T and P, Q, R.

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A Proof of Proposition 1

First, let us prove that if $\partial g(X,Y)/\partial X > 0$ holds at $(X,Y) \in \partial D$, $\partial S_{(X,Y)}$ circumscribes ∂T. With $(x,y) \in \partial T$, the curvature κ on ∂T is expressed as

$$\kappa = \frac{\mathrm{d}^2 y/\mathrm{d}x^2}{(1+(\mathrm{d}y/\mathrm{d}x)^2)^{3/2}} \tag{3}$$

$$= \frac{\ddot{Y}}{1 - Y\ddot{Y} - \dot{Y}^2}. \tag{4}$$

where we define $\dot{Y} = \mathrm{d}Y/\mathrm{d}X$, $\ddot{Y} = \mathrm{d}^2 Y/\mathrm{d}X^2$, and utilize $\mathrm{d}y/\mathrm{d}x = \dot{Y}/\sqrt{1-\dot{Y}^2}$, $\mathrm{d}^2 y/\mathrm{d}x^2 = \frac{\ddot{Y}}{(1-\dot{Y}^2)^{3/2}(1-Y\ddot{Y}-\dot{Y}^2)}$, which are derived [2]. Here, the condition that $\partial S_{(X,Y)}$ circumscribes ∂T is that $\kappa > -1/Y$ holds because a curvature of $\partial S_{(X,Y)}$ should be minus at $y > 0$. If $\partial g(X,Y)/\partial X > 0$ holds, this condition is expressed as $1 - (\mathrm{d}Y/\mathrm{d}X)^2 > 0$. This equation should hold because y is a real number in IBST. Therefore, the previous proposition is proved. We utilize this proposition to prove the proposition 1 as follows.

(a) Proof of $\partial S_+ \subset \partial T$.
We assume that a point $P = (x_\mathrm{p}, y_\mathrm{p}), (x_\mathrm{p} \in \gamma)$ exists, which satisfies $P \in \partial S_+,\ P \cap \partial T = \phi$, where ϕ is a null set. We define $Q = (x_\mathrm{q}, y_\mathrm{q}),\ (x_\mathrm{q} \in \gamma)$ on ∂T which satisfies that its distance from P is minimum of all points on ∂T. We define $(X_\mathrm{q}, Y_\mathrm{q})$ which is transformed from $(x_\mathrm{q}, y_\mathrm{q})$ with the BST, and $R = (X_\mathrm{q}, 0)$. Fig. 6 shows the relationship between ∂T and P, Q, R.

We define $x_{\min} = g(X_{\min}, Y_{\min})$ and $x_{\max} = g(X_{\max}, Y_{\max})$ on ∂T. Here $X_{\min} < X_\mathrm{q} < X_{\max}$ holds because $\partial g(X,Y)/\partial X > 0$ and $x_{\min} < x_\mathrm{q} < x_{\max}$ satisfies. Here, all $\partial S_{(X,Y)}$ on ∂D circumscribes ∂T because $\partial g(X,Y)/\partial X > 0$ holds. Therefore, $P \in S_\mathrm{q}$ satisfies because $\overline{PR} < \overline{QR}$ holds as shown in Fig. 6. $P \in S_+$ holds because of $S_\mathrm{q} \subset S_+$. Accordingly, $P \cap \partial S_+ = \phi$ holds because $\partial S_+ \cap S_+ = \phi$ satisfies. However, this equation contradicts to the previous assumption. Therefore $\partial S_+ \subset \partial T$ should hold.

(b) Proof of $\partial T \subset \partial S_+$.
We assume that $P = (x_\mathrm{p}, y_\mathrm{p})$ will exist where $P \in \partial T,\ P \cap \partial S_+ = \phi$ holds. With the IBST, $(x_\mathrm{p}, y_\mathrm{p})$ is transformed to $(X_\mathrm{p}, Y_\mathrm{p})$ where

$$(x_\mathrm{p} - X_\mathrm{p})^2 + y_\mathrm{p}^2 = Y_\mathrm{p}^2, \tag{5}$$

satisfies. For all $(X,Y) \in \partial D$, $(x_\mathrm{p} - X)^2 + y_\mathrm{p}^2 \geq Y^2$ holds because all $\partial S_{(X,Y)}$ on ∂D circumscribe ∂T. Therefore, $P \cap S_+ = \phi$ satisfies. If $P \cap \partial S_+ = \phi$ holds, $(x_\mathrm{p} - X)^2 + y_\mathrm{p}^2 > Y^2$ should hold in any $(X,Y) \in \partial D$. However, this fact contradicts Eq. (5). Therefore the assumption is not true, and $\partial T \subset \partial S_+$ is proved. According to the fact (a),(b), proposition 1 is proved.