An Estimation Method of Target Location and Scattered Waveforms for UWB Pulse Radar Systems

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Abstract— **This paper presents a method of estimating target location and scattered waveforms, whose accuracies are interdependent. The technique relies on iterative improvements of estimated dominant-frequency waveforms. Description of the algorithm is followed by statistical simulation examples. The performance of the technique is contrasted with conventional methods and statistical bounds in terms of target locationing accuracy. Results indicate that our proposed method has a remarkable performance, which is close to the theoretical limit.**

Keywords— **Super-resolution, UWB, Pulse radar, Radar imaging, Waveform estimation, Non-parametric**

I. Introduction

ENVIRONMENT measurement is an important issue for various applications including household robots. Radars utilizing ultra-wide-band (UWB) pulses, for which FCC has recently set a standard, are promising candidates in a near future. We have developed imaging algorithms for pulse radar systems based on the model fitting[1], which require a good initial guess of the location of targets. Target locationing using non-parametric algorithms can be used to provide an initial guess. Although non-parametric target locationing algorithms have also been proposed[2], many of conventional algorithms have a poor ranging accuracy compared with their estimation accuracy of DOA (Direction Of Arrival). Our objective is to develop a non-parametric super-resolution target locationing algorithm by improving the ranging accuracy iteratively.

An appropriate filtering is essential for a precise ranging. However, accurate noise reduction with Wiener filter requires information of the scattered waveform. On the other hand, target location estimation is indispensable to the waveform estimation. Therefore, it is required to estimate target locations and scattered waveforms simultaneously. In this paper, we propose a method which simultaneously estimates target locations and scattered waveforms for UWB pulse radar systems with array sensors. Moreover, we examine the performance of our method by contrasting it with conventional methods and statistical bounds using numerical simulations.

II. System Model

We assume an M-element linear sensor array with intervals of half-wavelength at the center frequency of the pulse, and one point target located within its near field. Fig. 1 shows the location of the sensor array and the coordinates, where λ is the center wavelength of the transmitted signals. We also define $\boldsymbol{T} = [T_x, T_y]$ as the real target location. The transmitted pulse is a mono-cycle pulse, which is suitable for radar systems because it has no DC power. We assume the received waveform is the 1st order differential of the transmitted waveform. The scattered wave is a spherical wave because the target is within the near field. Therefore, the signal delay draws a hyperbola as a function of the location of the sensors. We assume that the observer has no information of scattered waveforms.

We deal with a 2-dimensional problem in the paper. We also define a signal image $s(x, y)$ as

$$s\left((m-(M-1)/2)\,d/\lambda, ct/\lambda\right) \equiv s'_m(t), \qquad (1)$$

where $s'_m(t)$ is the received signal with the m-th sensor, c is speed of the light, and $d = \lambda/2$. This definition of a signal image is advantageous because space x and time y are normalized by wavelength. Our algorithm estimates the target location $\boldsymbol{T}$ using the signal image $s(x, y)$. Table 1 shows the simulation parameters.

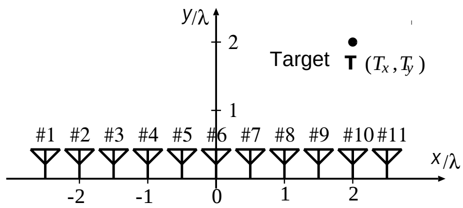

Fig. 1. The location of the sensor array and coordinates used in the present paper.

TABLE I
Simulation Parameters.

Sensor Array	$M = 11$
Sensor Interval	0.5λ
IHCT Iteration	40times
Observation Duration	24λ
Sampling	83 samples/λ

III. Waveform and Filtering

In this section, we explain the importance of estimating waveforms in the proposed algorithm. Wiener filter is often used for estimation of the turn-around-time because it is an effective denoising filter. Wiener filter for signal $G(\omega)$ is expressed as

$$W(\omega) = \frac{G^*(\omega)}{(1-\eta) + \eta|G(\omega)|^2}, \qquad (2)$$

where $\eta = 1/(1+(\mathrm{S/N})^{-1})$. $W(\omega)$ works as an inverse filter for large S/N ($\eta \simeq 1$). On the other hand, it works as a matched filter for small S/N ($\eta \simeq 0$). Here, we define the signal power $S = \max|s(x,y)|^2$. $W(\omega)$ is the optimal filter, so it minimizes the mean square error between the output signal and the impulse function. However, we can not directly apply Wiener filter to our purpose, because $W(\omega)$ requires the scattered waveform $G(\omega)$. This is the reason why our proposed method is important.

IV. Theoretical Limit of Estimation

In this section we derive the theoretical limit for our problem. The derived theoretical limit is based on Cramer-Rao lower bound (CRLB). We define $R_{\boldsymbol{T}-\boldsymbol{T}_i}$ as the covariance matrix of the estimation error of the target location. The original expression of CRLB is

$$R_{\boldsymbol{T}-\boldsymbol{T}_i} \geq J^{-1}(\boldsymbol{T}), \tag{3}$$

where $J(\boldsymbol{T})$ is Fisher information matrix expressed as

$$J(\boldsymbol{T})_{m,n} = -\mathrm{E}\left\{\iint \frac{\partial^2 \log p\,(s|\boldsymbol{T})}{\partial T_j \partial T_k}\mathrm{d}x\mathrm{d}y\right\}, \tag{4}$$

where $p(s|\boldsymbol{T})$ is the conditional probability density function of $s(x,y)$ and $j,k \in \{x,y\}$. We can not directly use Eq.(3) because the estimation error is expressed as $e_i = |\boldsymbol{T}-\boldsymbol{T}_i|$. We thus define $q(\Delta\boldsymbol{T})$ as the probability density function of $\Delta\boldsymbol{T} = \boldsymbol{T}-\boldsymbol{T}_{\mathrm{e}}$, where $\boldsymbol{T}_{\mathrm{e}}$ is the theoretically best estimation. We assume $q(\Delta\boldsymbol{T})$ as

$$q(\Delta\boldsymbol{T}) = \frac{(\det J(\boldsymbol{T}))^{1/2}}{2\pi}\exp\left[-\frac{1}{2}\Delta\boldsymbol{T}J(\boldsymbol{T})\Delta\boldsymbol{T}^{\mathrm{T}}\right]. \tag{5}$$

Assuming Eq.(5) gives

$$e_i \geq e_{\mathrm{CRLB}} = \int_{-\infty}^{\infty} |\Delta\boldsymbol{T}| q(\Delta\boldsymbol{T})\mathrm{d}\Delta\boldsymbol{T}. \tag{6}$$

e_{CRLB} is the theoretical limit for the estimation of target location. We calculate e_{CRLB} for each S/N in order to contrast with the simulation results. We call e_{CRLB} as CRLB for simplicity in the following sections.

V. The Proposed Method

In this section, we explain the proposed algorithm. We define the estimated target location for i-th iteration as $\boldsymbol{T}_i = (x_i, y_i)$. We define Hyperbolic Coherent Transform (HCT) as

$$H(\omega,\boldsymbol{T}_i) \equiv \iint_{-\infty}^{\infty} s(x,y)\frac{\mathrm{e}^{\mathrm{j}\omega[u(x,\boldsymbol{T}_i)-y]}}{\sqrt{u(x,\boldsymbol{T}_i)}}\mathrm{d}x\mathrm{d}y, \tag{7}$$

where we define

$$u(x,\boldsymbol{T}_i) \equiv |\boldsymbol{T}_i| + \sqrt{(x-x_i)^2+y_i^2}. \tag{8}$$

HCT works as the Fourier transform for y. $u(x,\boldsymbol{T}_i)$ is a delay time compensation for x. $\sqrt{u(x,\boldsymbol{T}_i)}$ is required in order to improve S/N of HCT. HCT estimates $F(\omega)$, which is the Fourier transform of the scattered waveform, using coherent integral of the received signals. We can describe the algorithms of target location estimation as

$$\mathrm{maximize}_{\boldsymbol{T}_{i+1}} \left|\int_{-\infty}^{\infty} \frac{H(\omega,\boldsymbol{T}_{i+1})P_i^*(\omega)}{1-\eta+\eta|P_i(\omega)|^2}\mathrm{d}\omega\right|^2, \tag{9}$$

where $P_i(\omega)$ is the waveform used for constructing Wiener filter. Eq.(9) includes all algorithms we investigate in the paper, which depends on the definition of $P_i(\omega)$. We set the initial waveform $H(\omega,\boldsymbol{T}_0)$ as the Fourier transform of the transmitted waveform. We optimize Eq.(9) using Quasi-Newton Method algorithm, where we set the initial value of $\boldsymbol{T}_i$ to the optimized $\boldsymbol{T}_{i-1}$. We determine the initial value of $\boldsymbol{T}_1$ using a simple grid search.

We set $P_i(\omega)$ to

$$P_i(\omega) = (H(\omega,T_i) * \mathrm{sinc}(t_0\omega))\,|P_{i-1}(\omega)| \tag{10}$$

for the proposed algorithm. We call the proposed algorithm IHCT (Iterative HCT) because it is based on iterative improvement of estimation. Eq.(10) works as extraction of dominant-frequency waveform. Convolution of $\mathrm{sinc}(t_0\omega)$ is a simple windowing, which prevent the waveform from having extremely narrow band. We set t_0 to the pulse duration of the transmitted signal. Fig.2 shows the outline of IHCT. We also define IHCTW (IHCT Without waveform estimation) which is a conventional method. We set $P_i(\omega)$ for IHCTW as $P_i(\omega) = H(\omega,\boldsymbol{T}_0)$, which is the transmitted waveform. Moreover, we investigate IHCTK (IHCT with Known scattered waveform) which represents the ideal situation. We set $P_i(\omega)$ for IHCTK as $P_i(\omega) = F(\omega)$, which is the true scattered waveform. IHCTK is not realistic because $F(\omega)$ is unknown in an actual case. Table 2 shows $P_i(\omega)$ for each method.

TABLE II
$P_i(\omega)$ (Denoised HCT) for each method.

IHCT	$(H(\omega,T_i) * \mathrm{sinc}(t_0\omega))\,|P_{i-1}(\omega)|$
IHCTW	$H(\omega,T_0)$
IHCTK	$F(\omega)$

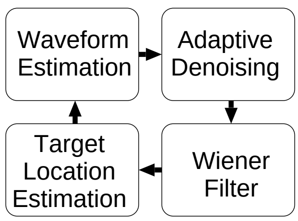

Fig. 2. The outline of IHCT.

VI. Performance Evaluation

In this section we investigate the performance of the proposed method by contrasting with the conventional method and the theoretical limit. Fig.3 illustrates the waveform of $P_i(\omega)$ for $i = 1, 5$ and 10. The bandwidth of the waveform becomes narrower as the iteration proceeds.

Fig.4 shows the locationing accuracy of each algorithm compared to CRLB. Here, we set the target location to $\boldsymbol{T} = (2\lambda, 2\lambda)$. The relationship between the estimation error e_L and the peak S/N is illustrated in the figure. IHCT, IHCTW and IHCTK have poor performance for S/N $<$ 11dB due to invalid initial guess of $\boldsymbol{T}_1$, which is caused by small S/N. IHCTK achieves CRLB for S/N $\geq$ 11dB, which means the optimization in Eq.(9) can achieve the theoretical limit only if we know the scattered waveform $F(\omega)$. IHCTW has a floor of estimation error for S/N $\geq$ 11dB, which is caused by biases due to the fixed reference waveforms. The difference between the transmitted waveform and the scattered waveform causes this error. On the other hand, the performance of IHCT is close to CRLB. The ratio of the estimation accuracy of IHCT to that of CRLB is 1/4 at most. The estimation error of IHCT has no floor for S/N $\leq$ 40dB. The estimation accuracy of IHCT is 140 times better than that of IHCTW. Moreover IHCT achieves an accuracy of $10^{-3}\lambda$ for S/N $>$ 34dB.

Fig.5 shows the estimation error of target location using IHCT for various target locations for S/N = 40dB. From the figure, we see that the order of estimation error is $10^{-3}\lambda$ for all target location except for the two areas on both sides of the array. The poor performance of IHCT in the two areas is caused by the ambiguity of the signal with target locations.

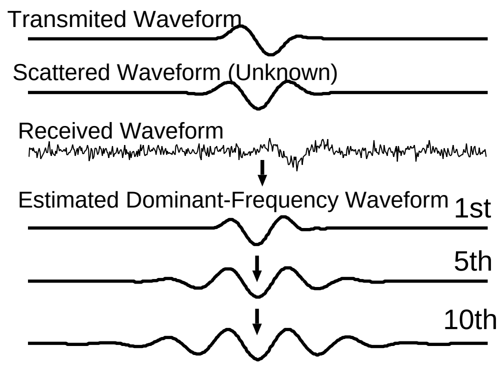

Fig. 3. Estimated dominant-frequency waveforms.

VII. Summary

UWB pulse radar sysmtes are promising candidates for environment measurement. In the present paper, we proposed a high-resolution algorithm for target locationing without information of scattered waveforms. The proposed method simultaneously estimates target locations and scattered waveforms for UWB pulse radar systems. The proposed method estimates dominant-frequency waveforms of scattered waveform iteratively. We also examined the performance of our method by contrasting them with conventional methods and statistical bounds. We evaluated the performance in terms of the estimation accuracy of target locations utilizing numerical simulations. We showed that the performance of the proposed method is close to the theoretical limit. We clarified that the estimation accuracy of the proposed method is 140 times better than that of the conventional method. We also made it clear that the proposed method achieves an accuracy of $10^{-3}\lambda$ for S/N $>$ 34dB.

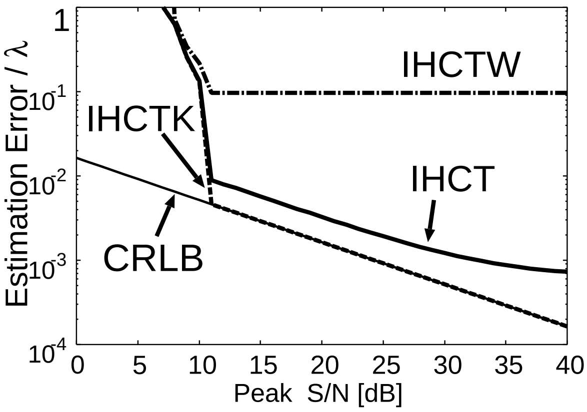

Fig. 4. Estimation error of the target location.

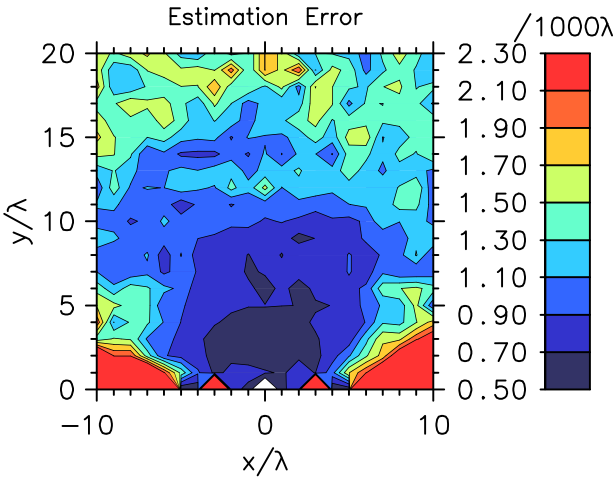

Fig. 5. Estimation error for various target locations.

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